



# MONOGRAPHS ON PHYSICS

EDITED BY

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MASTER OF TRINITY COLLEGE, CAMBRIDGE

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GALVANOMAGNETIC AND THERMOMAGNETIC EFFECTS: THE HALL AND ALLIED PHENOMENA. By L. L. CAMPBELL, Ph.D., Professor of Physics, Simmons College, Boston, U.S.A.

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# GALVANOMAGNETIC AND THERMOMAGNETIC EFFECTS

THE HALL AND ALLIED PHENOMENA

BY

L. L. CAMPBELL, Ph. D.

PROFESSOR OF PHYSICS, SIMMONS COLLEGE, BOSTON, U.S.A.

*WITH DIAGRAMS*

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1923



This monograph is gratefully dedicated to

EDWIN HERBERT HALL.

Who opened to the scientific world a vast and fascinating domain, wherein dwell some of the mysteries of matter; who by his works enriched science; and who by his life added new honour and dignity to man.

THE AUTHOR.

## PREFACE

THE purpose of this monograph is to bring together the historical, experimental and theoretical accounts of that family of galvanomagnetic and thermomagnetic phenomena that are the lineal offspring of the Hall effect.

The fact that the several effects have different signs in different metals has thus far received no thoroughly satisfactory explanation. The compilation of the results of the researches on these effects, and the statement of the leading theories concerning the phenomena, it is hoped, may prove a step toward removing the one great obstacle in the way of a satisfactory application of the electron theory to the electrical and thermal phenomena in metals.

The following paragraph from Hall's original paper <sup>(235)</sup> suggests some things that may be expected from a thorough and complete study of these effects in all metals:

"On the whole we cannot be sure that any relation has yet been detected between the magnitude of the new action in the various metals and any known physical property of those metals. It is of course possible, however, that when more data shall have been obtained, analogies and relations at present unsuspected will appear. It can hardly be doubted that the action we have been considering, placing at our command as it does, a new point of view from which to study the interior workings of the substances examined, is destined to teach us a good deal in regard to the molecular structure

of the bodies, while helping us toward an understanding of the physical nature of electricity and magnetism."

The investigations herein treated have already done much to cast light on the inter-atomic and intra-atomic actions in matter; yet much remains to be done to secure a reconciliation of the proposed theories and the known facts.

L. L. CAMPBELL.

*Cambridge, Mass., 1923.*

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# GALVANOMAGNETIC AND THERMOMAGNETIC EFFECTS

## INTRODUCTION

### A. GALVANOMAGNETIC EFFECTS

If a metal plate, carrying a current of electricity,  $I$ , Fig. 1, is placed in a magnetic field,  $H$ , so that the lines of magnetic force are at right angles to the plane of the plate and to the direction of the flow of the electric current, two transverse and two longitudinal effects may be observed in the metal, as follows:

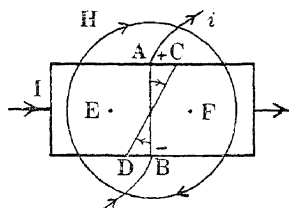


Fig. 1.

1. A transverse magnetic potential difference, or a rotation of the equipotential lines.

For example, the rotation of the equipotential line  $AB$  to a new position  $CD$ , Fig. 1. This is the original *Hall Effect*.

2. A transverse galvanomagnetic temperature difference at  $A$  and  $B$ , Fig. 1: the *Ettingshausen Effect*.

3. A longitudinal galvanomagnetic potential difference between such points as  $E$  and  $F$ , Fig. 1: a change in electrical resistance.

4. A longitudinal galvanomagnetic temperature difference between such points as  $E$  and  $F$ , Fig. 1.

### B. THERMOMAGNETIC EFFECTS

If a metal, through which is flowing a current of heat, is placed in a magnetic field,  $H$ , Fig. 2, so that the lines of magnetic force are perpendicular to the plane of the plate and to the direction of flow of the heat current, two transverse and two longitudinal effects, analogous to the galvanomagnetic effects, may be observed, as follows:



1. A transverse thermomagnetic potential difference between *A* and *B*: the *Nernst effect*.

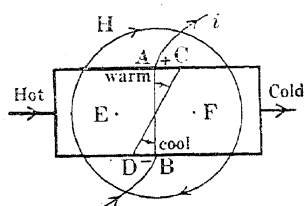


Fig. 2.

2. A transverse thermomagnetic temperature difference between *A* and *B*, or the rotation of the isothermal line *AB*: the *Righi-Leduc Effect*.

3. A longitudinal thermomagnetic potential difference between such points as *E* and *F*.

4. A longitudinal thermomagnetic temperature difference between *E* and *F*: a change in thermal conductivity.

The table below exhibits in more concise form the relations of the several effects. The literal-descriptive coefficients were proposed by Hall <sup>(247)\*</sup>.

*Effects Produced by a Transverse Magnetic Field*

	Galvanomagnetic		Thermomagnetic	
	Electro-Electric	Electro-Thermal	Thermo-Electric	Thermo-Thermal
Transversal Effects	Difference of Potential: Hall Effect, $eT_e$	Difference of Temperature: Ettingshausen Effect, $eTh$	Difference of Potential: Nernst Effect, $hT_e$	Difference of Temperature: Righi-Leduc Effect $hTh$
Longitudinal Effects	Difference of Potential: Change in Resistance, $eLe$	Difference of Temperature, $eLh$	Difference of Potential, $hLe$	Difference of Temperature: Change in Thermal Conductivity, $hLh$
Kindred Phenomena	Electromotive Force of Magnetization	Change in Peltier Effect in Magnetic Field	Change in Thermoelectric Property in Magnetic Field	Change in Temperature in Magnetic Field

Some of the above effects have also been observed in a longitudinal magnetic field.

\*) The numbers in the elevated parentheses refer to titles of articles in the Bibliography, pp. 273—308.

# A. GALVANOMAGNETIC PHENOMENA

## CHAPTER I

### THE HALL EFFECT: TRANSVERSE GALVANOMAGNETIC POTENTIAL DIFFERENCE

#### 1. Historical.

Sir Wm. Thomson (Lord Kelvin) in his paper on the *Dynamical Theory of Heat*<sup>(530)</sup>, 1851, first predicted the possibility of a "rotatory power" in a metal in a magnetic field. In paragraphs added in 1854, occur these statements:

"It is far from improbable that a piece of *iron* in a state of magnetization, which I have ascertained to possess different thermo-electric properties in different directions, may also possess rotatory thermo-electric power. By an experiment made to test its existence, which has given only negative results, I have ascertained that this 'rotatory power', if it exists in inductively magnetized iron at all, must be very small in comparison with the amount by which the thermo-electric power in the direction of magnetization differs from the thermo-electric power of the same metal not magnetized."

Feilitzsch<sup>(195)</sup>, in 1858, made an attempt to find out whether a magnetic field produced any effect on a current of electricity. He connected to a galvanometer two coils of wire. The electric currents in these coils were in opposite directions, and were so adjusted that they caused no deflection in the galvanometer. Near one of the coils he brought a third coil carrying a current of electricity, and in conse-

quence setting up a magnetic field. No permanent deflection of the galvanometer was noticed, and no permanent action of one current on the other was detected.

Mach <sup>(350)</sup>, in 1870, endeavored to discover the effect of an electromagnet on a current of electricity. He used a circular disk of *silver* covered with wax. A current of electricity was sent through a diameter of the disk. The silver leaf became heated and the wax melted. The plate was then placed in a magnetic field to see if the melting became unequal on the two sides of the diameter, due to the shifting of the current. The condition of the melted wax remained unchanged in the magnetic field.

In 1874, Gore <sup>(216)</sup> made the following experiment to see if a current of electricity was affected, or deflected, by a magnetic field:

"I passed a powerful voltaic current through the thick copper wire of a large electromagnet, and then divided it equally between two vertical pieces of thin *platinum* wire of equal diameter and length (about six or seven centimeters), so as to make them equally white hot, the two wires being attached to two horizontal cross wires of copper.

"On approaching the two vertical wires symmetrically towards the vertical face of one pole of the horizontally placed magnet, and at equal distances from it, so that the two downward currents in them might be equally acted upon by the downward and upward portions respectively of the currents which circulated round the magnet-pole, the one was strongly bent towards and the other from the pole, as was, of course, expected, but not the least sign of alteration of relative temperature of the two wires could be perceived, thereby proving that not even a small proportion of the current was repulsed from the repelled wire or drawn into the attracted one, as would have occurred had the attraction and repulsion taken place, even to a moderate degree, between the currents themselves; and I therefore conclude that the

*attractions and repulsions of electric conductors are not exerted between the currents themselves, but between the substances conveying them."*

The failure of Feilitzsch, Mach, and Gore to detect the effect they sought was due to the fact that the methods employed were crude, to the fact that the effect is due to a secondary electromotive force, almost infinitesimally small compared with the primary electromotive force, and to the lack of sensitive galvanometers.

In 1879, E. H. Hall <sup>(234)</sup>, then a Fellow of Johns Hopkins University, published an article entitled, "On a New Action of the Magnet on Electric Currents." As a matter of historical interest, it seems worth while to quote several paragraphs from this paper.

"Sometime during the last University year (1878), while I was reading Maxwell's Electricity and Magnetism in connection with Professor Rowland's lectures, my attention was particularly attracted by the following passage in Vol. II, p. 144: 'It must be carefully remembered, that the mechanical force which urges a conductor carrying a current across the lines of magnetic force, acts not on the electric current, but on the conductor which carries it. If the conductor be a rotating disk or fluid it will move in obedience to this force, and this motion may or may not be accompanied with a change of position of the electric current which it carries. But if the current itself be free to choose any path through a fixed solid conductor or network of wires, then when a constant magnetic force is made to act on the system, the path of the current through the conductors is not permanently altered, but after certain transient phenomena, called induction currents, have subsided, the distribution of the current will be found to be the same as if no magnetic force were in action. The only force which acts on electric currents is electromotive force, which must be distinguished from the mechanical force which is the subject of this chapter.

"This statement seemed to me contrary to the most natural supposition in the case, taking into account the fact that a wire bearing a current is affected exactly in proportion to the strength of the current, while the size and, in general, the material of the wire are matters of indifference."

"Soon after reading the above statement in Maxwell I read an article by Prof. Edlund <sup>(16t)</sup>, entitled '*Unipolar Induction*', in which the author evidently assumes that a magnet acts upon a current in a fixed conductor just as it acts upon the conductor itself when free to move.

"Finding these two authorities at variance, I brought the question to Prof. Rowland. He told me he doubted the truth of Maxwell's statement and had some time before made a hasty experiment for the purpose of detecting, if possible, some action of the magnet on the current itself, though without success. Being very busy with other matters, however, he had no immediate intention of carrying the investigation further.

"I now began to give the matter more attention and hit upon a method that seemed to promise a solution of the problem. I laid my plan before Prof. Rowland and asked whether he had any objection to my making the experiment. He approved of my method in the main, though suggesting some very important changes in the proposed form and arrangement of the apparatus. The experiment proposed was suggested by the following reflection:

"If the current of electricity in a fixed conductor is itself attracted by a magnet, the current should be drawn to one side of the wire, and therefore the resistance experienced should be increased.

"To test this theory, a flat spiral of *German silver* wire was inclosed between two thin disks of hard rubber and the whole placed between the poles of an electromagnet in such a position that the lines of magnetic force would pass through the spiral at right angles to the current of electricity."

After describing a series of observations, the author remarks: "Apparently, then, the magnet's action caused no change in the resistance of the coil."

"But though conclusive, apparently, in respect to any change of resistance, the above experiments are not sufficient to prove that a magnet cannot affect an electric current. If electricity is assumed to be an incompressible fluid, as some suspect it to be, we may conceive that the current of electricity flowing in a wire cannot be forced into one side of the wire or made to flow in any but a symmetrical manner. The magnet may *tend* to deflect the current without being able to do so. It is evident, however, that in this case there would exist a state of stress in the conductor, the electricity pressing, as it were, toward one side of the wire. Reasoning thus, I thought it necessary, in order to make a thorough investigation of the matter, to test for a difference of potential between points on opposite sides of the conductor.

"This could be done by repeating the experiment formerly made by Prof. Rowland, and which was the following:

"A disk or strip of metal, forming a part of an electric circuit, was placed between the poles of an electromagnet, the disk cutting across the lines of force. The two poles\* of a sensitive galvanometer were then placed in connection with different parts of the disk, through which an electric current was passing, until two nearly equipotential points were found. The magnet current was then turned on, and the galvanometer was observed, in order to detect any indication of a change in the relative potential of the two poles.

"Owing probably to the fact that the metal disk used had considerable thickness, the experiment at that time failed to give any positive result. Prof. Rowland now advised me, in repeating this experiment, to use *gold leaf* mounted on a plate of glass as my metal strip. I did so, and experimenting as indicated above, succeeded on the 28<sup>th</sup> of October (1879)

in obtaining, as the effect of the magnet's action, a decided deflection of the galvanometer needle.

"This deflection was much too large to be attributed to the direct action of the magnet on the galvanometer needle, or to any similar cause. It was, moreover, a permanent deflection and therefore not to be accounted for by induction.

"The effect was reversed when the magnet was reversed. It was not reversed by transferring the poles of the galvanometer from one end of the strip to the other. In short, the phenomena observed were just such as we should expect to see if the electric current were pressed, but not moved, toward one side of the conductor."

"It is perhaps allowable to speak of the action of the magnet as setting up in the strip of gold leaf a new electromotive force at right angles to the primary electromotive force.

"This new electromotive force cannot, under ordinary conditions, manifest itself, the circuit in which it might work being incomplete. When the circuit is completed by means of the Thomson galvanometer, a current flows."

Shortly after Hall's discovery, Rowland <sup>(462)</sup>, 1880, wrote as follows:

"The recent discovery by Mr. Hall of a new action of magnetism on electric currents opens a wide field for the mathematician, seeing that we must now regard most of the equations which we have hitherto used in electromagnetism as only approximate, and as applying only to some ideal substance which may or may not exist in nature, but which certainly does not include ordinary metals. But as the effect is very small, probably it will always be treated as a correction to the ordinary equations."

## 2. The Hall Coefficient

The total transverse electromotive force,  $\mathcal{E}$ , set up by a magnetic field,  $H$ , in a plate  $P$ , Fig. 3, carrying an electric current  $I$ , is proportional to the width of the plate  $b$ , to the

strength of the field  $H$ , and to the current density,  $I_a$ . The electromotive force per unit width is therefore  $\frac{E}{b} = RHI_a$ , in which  $R$  is a constant depending on the metal, and is called the Hall coefficient.

If  $d$  is the thickness of the plate, then the current density  $I_a = \frac{I}{bd}$ . Substituting this in the above equation, we have  $E = \frac{RHI}{d}$ , and  $R = \frac{Ed}{IH}$ , in which  $E$  and  $I$  are in absolute units,  $d$  in cms., and  $H$  in gauss. The Hall coefficient,  $R$ , may be said to be the transverse difference of potential set up by a unit field in a plate 1 cm. thick and carrying a unit current.

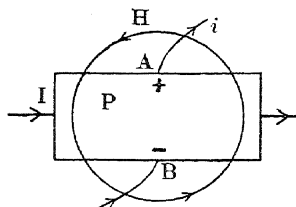


Fig. 3.

For example, suppose a *gold* plate 0.01 cm thick, carrying a current of 2 amperes, or 0.2 abs. units, is placed in a field of 10,000 gauss, that sets up a total transverse E. M. F. of 0.00000140 volt, or 140 abs. units. Then we shall have for the Hall coefficient in gold,

$$R = \frac{140 \times 0.01}{10,000 \times 0.2} = 0.0007 \text{ Abs. E. M. F. per}$$

gauss, per abs. unit current, per cm. thickness.

### 3. The Sign of the Hall Coefficient

The Hall effect may be considered as due to the rotation by the magnetic field of the equipotential lines in a plate carrying an electric current. Hall, in the researches already referred to, found that in *iron, cobalt, zinc, and antimony* the lines were rotated in one direction; while in *gold, silver, nickel, bismuth, tin, copper, platinum, aluminium, and magnesium* the lines were rotated in the opposite direction.

The following Figs. 4 to 9 represent the convention introduced by Hall as to the positive and negative Hall effects in metals.



The plate is represented by the rectangle  $P$ , the circle  $H$  represents the magnetic field, the direction of the Amperian current in the electromagnet is given by the arrowhead,  $I$  is

*Positive Hall Coefficient*

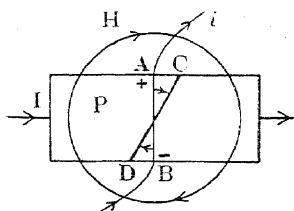


Fig. 4.

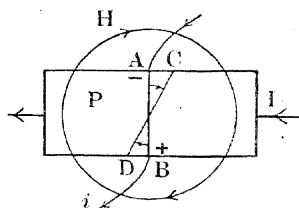


Fig. 5.

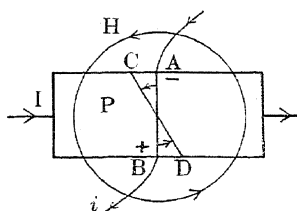


Fig. 6.

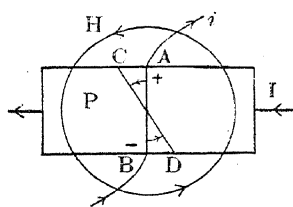


Fig. 7.

*Negative Hall Coefficient*

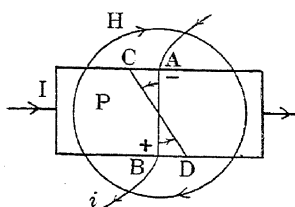


Fig. 8.

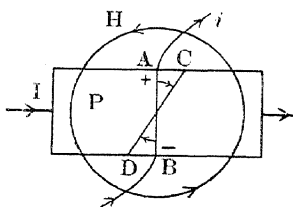


Fig. 9.

the primary current through the plate,  $A$  and  $B$  are the points to which the secondary leads are attached,  $AB$  is an equipotential line before the field  $H$  is on, and  $CD$  the position of this equipotential after the field has been put on.

If, as in Figs. 4, 5, 6, and 7, the equipotential line  $CD$  is rotated in the direction of the Amperian current of the

magnetic field  $H$ , the effect is said to be *positive*. This was found to be the case in *iron*, *cobalt*, and *antimony*.

If the equipotential line  $CD$  is rotated in a direction opposed to the Amperian current  $H$ , as in Figs. 8 and 9, the effect is said to be *negative*. According to this convention, the effect in *gold*, *nickel*, and *bismuth* is negative.

The reversal of the primary current  $I$  does not reverse the rotation of the equipotential line, as shown in Figs. 4 and 5. The direction of the secondary current  $i$ , is however, reversed.

The reversal of the field  $H$ , Figs. 4 and 6, reverses both the rotation of the equipotential and the direction of the secondary current,  $i$ .

#### 4. Angle of Rotation of Equipotential Lines

Suppose that Fig. 10 represents a metal plate,  $P$ , in which the primary current  $I$  enters at  $L$  and leaves at  $N$ . The current will spread out somewhat as indicated by the curved stream lines. Suppose that points  $A$  and  $B$  are at the same potential when the magnetic field is not on. When the field  $H$  is put on, the equipotential line  $AB$  will be rotated through some such angle as  $\Phi$  into a new position  $CD$ . The perpendicular  $CE$  will make with  $CD$  an angle  $\Phi$  also.

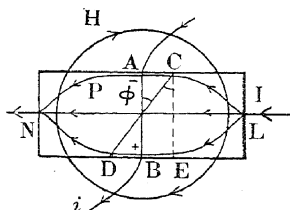


Fig. 10.

Then 
$$\tan \Phi = \frac{DE}{CE}.$$

If  $w$  is the width of the plate, and  $l$  the distance  $DE$ , then

$$\tan \Phi = \frac{l}{w}.$$

The difference of potential between  $E$  and  $D$  is

$$E = \frac{RH \times I}{d} \quad (\text{p. 9}).$$

By Ohm's law,  $E = r \times I$ ,

where  $r$  is the resistance and  $I$  the current in the plate from  $E$  to  $D$ .

Equating the values of  $E$ , we have

$$r \times I = \frac{RH \times I}{d},$$

and therefore

$$r = \frac{RH}{d}.$$

If  $\rho$  is the specific resistance of the metal in the plate, then

$$r = \frac{\rho l}{a},$$

where  $a$  is the area of cross-section of the plate. Equating the values of  $r$  and putting for  $a$  its equal  $w \times d$ , we obtain

$$\frac{\rho l}{w \times d} = \frac{RH}{d},$$

and

$$l = \frac{RHw}{\rho}.$$

Substituting this value of  $l$  in

$$\tan \Phi = \frac{l}{w},$$

we have

$$\tan \Phi = \frac{RH}{\rho} = R\sigma H,$$

where  $\sigma$  is the specific conductivity of the metal.

The meaning of this formula may become clearer, if it is applied to a concrete case.

The Hall coefficient of *gold* may be taken as 0.0007, the specific resistance of gold in absolute units is  $2.42 \times 10^3$ . Then a field of 10,000 gauss will rotate the equipotential line in gold through an angle given by the equation

$$\tan \Phi = \frac{0.0007 \times 10,000}{2.42 \times 10^3}.$$

Hence  $\Phi = 10'$  (approx.).

Since the coefficient of gold is negative, the equipotential line  $AB$  will be rotated  $10'$  in a direction opposed to that of the current in the electromagnet, as in Figs. 8 and 9.

In the uniform portion of the field, the stream lines of the primary current are not rotated nor distorted, hence the equipotential lines after rotation make an angle  $\frac{\pi}{2} - \Phi$  with the stream lines of the primary current, and are not perpendicular to the stream lines.

### 5. Primary Current Lines in the Hall Effect

While in the Hall effect the equipotential lines are rotated by the magnetic field, the lines of flow of the primary current are not rotated. In Fig. 11, the horizontal lines represent the

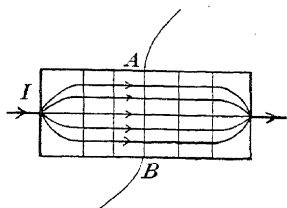


Fig. 11.

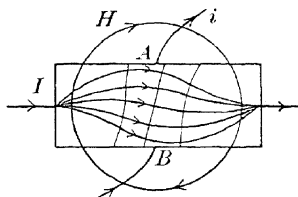


Fig. 12.

lines of flow of the primary current,  $I$ , in the plate when no field is on. The vertical lines are the equipotentials.

In Fig. 12, we have a diagram similar to those often found in articles on the Hall effect. In this both the stream lines and the equipotentials are rotated. This, however, is not the actual case.

In Fig. 13, we have a true representative of the behavior of the stream lines and the equipotentials, when the plate is in a uniform magnetic field  $H$ . The equipotentials are rotated as indicated by the inclined lines, while the stream lines of the primary current are not bent so as to become perpendicular to the equipotentials.

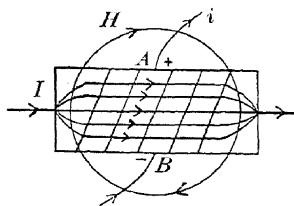


Fig. 13.

That the stream lines of the primary current should not be bent in a uniform magnetic field may be seen from the

following considerations suggested by Hall verbally to the writer.

Suppose that  $TT$  and  $BB$ , Fig. 14, represent the top and the bottom respectively of a horizontal pipe, through which is flowing a constant stream of water, under a constant head pressure.

If friction and gravity are neglected, the line  $AD$ , perpendicular to the stream lines, represents a line of equal pressure. The weight of the water, however, exerts a pressure that increases with depth, from  $A$  to  $D$ . The result of the pressure impressed on the water and of the gravity pressure will be that the equi-pressure line  $AD$  will be rotated to

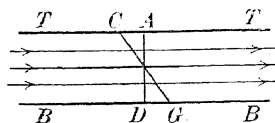


Fig. 14.

some such position as  $CG$ . The gravity pressure will slightly increase the density of the water toward the side  $BB$ , but the stream lines will not be bent so that they will be perpendicular to  $CG$ .

In water the current density, however, will be slightly increased on the lower side of the tube, water not being an incompressible fluid; but in the flow of electricity nothing of this kind is detected.

The following extracts from *Nature* give some account of remarks by Sir Wm. Thomson <sup>(536)</sup>, on the stream lines in the Hall effect, before the American Association for the Advancement of Science in 1884:

"Sir Wm. Thomson spoke of the discovery of Mr. Hall as being the most important made since the time of Faraday." ...

"He showed that the mathematical examination of the subject indicated three relations to be investigated: first, the relation of thermal force to the surfaces of equal rate variation of temperature; second, the relation of electric current to the equipotential surfaces; third, the relation of the thermal flow to isothermal surfaces. The second of these is that investigated by Mr. Hall, who has found that under the

conditions the lines of flow are *not* perpendicular to the equipotential surfaces. There remains, therefore, 'work for two more Halls', in either proving or disproving the existence of analogous actions in these two other cases."

"Sir Wm. Thomson also suggested the following exceedingly interesting mechanical illustration or analogue of Hall's effect. Let us be living upon a table which rotates uniformly forever. A narrow circular canal is upon the table, and nearly full of water. After a while the water will acquire the same velocity of rotation as the table, and will come to a state of equilibrium. The outer edge of the water in the canal will then stand a little higher than the inner edge. Let us now apply a little *motive* force to the water, and by means of a pump cause it to flow in the canal in the same direction in which the table is rotating: it is evident that it will stand higher on the outer edge, and lower on the inner edge of the canal, than before. But, should we cause it to flow in the opposite direction to the motion of the table, it will stand lower on the outer edge, and higher on the inner edge, than in its position of equilibrium."

Corbino <sup>(120)</sup>, 1915, in an article on the movement of electricity in a plate subjected to the action of a magnetic field perpendicular to the plane of the plate, maintained that the equipotential lines, or the stream lines of the primary current, or both, which originally formed an orthogonal system, are so deformed that an isoclinic system is produced throughout the plate.

Volterra <sup>(566)</sup>, 1915, and Righi <sup>(452)</sup>, 1915, followed Corbino's discussion with more elaborate mathematical treatment. They claimed that the current density is shifted by the action of the magnetic field on the primary current in the plate; in other words, that the electrons or ions are deflected and crowded toward one side of the plate, electrolyte, or discharge tube.

## CHAPTER II

## METHODS OF DETERMINING THE HALL EFFECT

## 1. Hall's Original Method

The following paragraphs from a second paper by Hall (<sup>235</sup>), 1880, will best give an account of the method followed in his classic experiment.

"On the same page of the 'Galvanismus' which treats of the research by Mach, as mentioned above, Wiedemann describes, as a means of showing that no action takes place

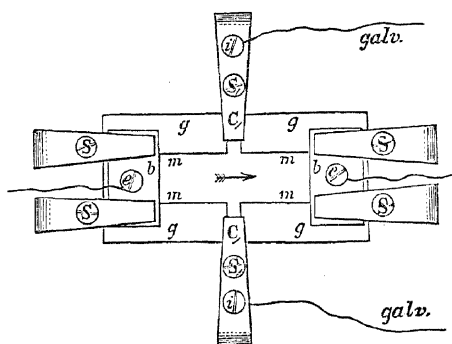


Fig. 15.

between permanent electric currents as such, almost the exact arrangement of apparatus with which the discovery was finally made. Who first used this apparatus for this purpose I cannot say, unless it may have been Wiedemann himself. The same plan was hit upon by Professor Rowland, quite independently I believe, and he experimented to some extent in this direction in 1876. The same arrangement was finally adopted by me, after another method of attacking the problem had been unsuccessfully tried."

The following is a description by Hall of the apparatus used by him in the discovery of the Hall effect. Figure 15 is a reproduction of the diagram given in his paper, but it is not a picture of the original plate:

"In Fig. 15, which is about one-half the actual size of an ordinary plate,  $g, g, g, g$  represents the plate of glass upon which the metal strip  $m, m, m, m$  is mounted. Contact with this strip is made at the ends by the two thick blocks of brass  $b, b$ , which are held firmly in place by the four brass clamps worked by means of the screws  $S, S, S, S$ . The main current of electricity enters and leaves the metal strip by means of the binding screws  $e, e$ . Running out from the middle of this strip are two projections which make contact with the clamps  $C_1, C_1$ , worked by the screws  $S_1, S_1$ . From the screws  $i, i$ , wires lead to the Thomson galvanometer. The projections from the metal strip just alluded to make the apparatus very easy to adjust, for by scraping off little particles from the proper part of the projections, while the current is allowed to run through the metal strip, the current through the Thomson galvanometer may be reduced to the extent desired.

"In ordinary experiments such a plate as that just described is placed between the poles of the magnet in such a position that the direction of magnetic force would be represented by a perpendicular to the plane of the paper in the above drawing.

"With this arrangement a permanent deflection of about 30 cm. on the scale before the Thomson galvanometer could be obtained by reversal of the magnetic current." . . . . "This experiment, then, shows our main fact, viz.: that in a conductor subjected to the given conditions a permanent electromotive force is at once established which has a direction perpendicular to the direction of the magnetic force and perpendicular to the direction of the primary current in the conductor."

## 2. Hall's Bath Method

To eliminate thermoelectric effects due to inequalities of temperature in the plate of metal, Hall <sup>(245)</sup>, 1885, used a tank filled with water, in which the plate was immersed. A hori-



horizontal cross-section of this tank,  $T, T$ , is shown in Fig. 16. The tank was placed between the poles  $P, P$ , of the electromagnet. Sometimes during the investigation water was kept running into and out of the tank, at other times the water remained at rest.

The use of the tank necessitated a different method of sending the primary current through the plate. The current  $I$  was led to and from the plate by means of brass strips  $B, B$ , Fig. 17. These strips were soldered to the plate. Small wires,

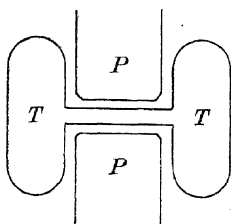


Fig. 16.

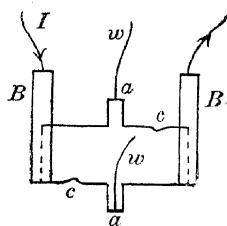


Fig. 17.

$w, w$ , were soldered to the arms,  $a, a$ . The notches  $c, c$ , were filed until the points  $a, a$ , were practically of the same potential, when the primary current was flowing and the magnetic field was not on. The metal and brass strips were fastened to a glass plate by means of a rosin and bees-wax cement, and the whole thoroughly covered with the cement and made water tight. The plate thus mounted was placed vertically in the tank between the poles of the electromagnet.

### 3. Righi's Method

Righi <sup>(444)</sup>, 1883, in determining the Hall effect, did away with the lateral electrodes used by Hall. He split the plate,  $P$ , as shown in Fig. 18. The primary current  $I$  was led by a wire into one end of the plate, and this current was led from the plate by two wires  $w, w$ , to a differential galvanometer,  $G$ . The divided current circulated in opposite directions in the galvanometer coils. The ends of the plate,  $A, B, C$ ,

were covered with tin foil, and to this were attached the lead wires.

By means of adjustable resistances the deflection of the galvanometer was made null when the magnetic field was

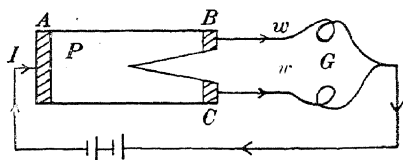


Fig. 18.

not on. When the magnetic field was thrown on, the current in one of the  $w$  wires was increased, while that in the other was diminished, and this difference was indicated by a deflection of the galvanometer. Ettingshausen <sup>(168)</sup> has shown that the deflection of the galvanometer indicated only one half of the true Hall effect.

Righi, using a plate of *bismuth* 0.079 mm. thick, found the Hall effect in that metal 5000 or 6000 times as great as that in gold. The effect was so large that when the plate was placed at right angles to the field of a permanent bar magnet, the galvanometer gave a permanent deflection of about 50 cm.

Righi mounted a plate of *bis-muth* as shown in Fig. 19, so that the plate and holder could be rotated in a horizontal and in a vertical plane, about a clamp  $N$ . When the plate was vertical and normal to the magnetic meridian, the earth's magnetic force caused a deflection of the galvanometer of about 9 mm. When the plate was inclined so as to be per-

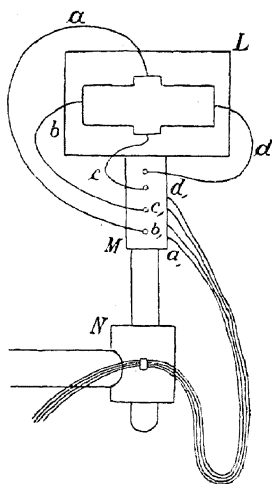


Fig. 19.

pendicular to the earth's magnetic lines of force, a deflection of about 12 mm. resulted.

In addition to the split plate, Righi used plates such as those indicated in Figs. 20 and 21. With those plates he

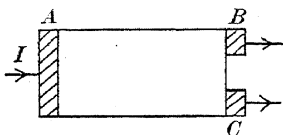


Fig. 20.

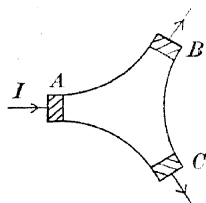


Fig. 21.

obtained results in accord with those found with the cruciform and split plates. Righi's method has the merit that small plates and plates of irregular shape may be used. The method, however, is probably not a very accurate one, and can be used to advantage only where the Hall effect is large.

#### 4. Leduc's Capillary Electrometer Method

Leduc<sup>(313)</sup>, in 1884, in determining the Hall effect, used a capillary electrometer in place of the ordinary galvanometer. The Hall electromotive force was determined by the method of compensation.

#### 5. Ettingshausen and Nernst Method

In addition to the regular Hall rectangular type of plate, Ettingshausen and Nernst<sup>(174)</sup>, 1886, used semi-circular and circular plates in their study of the Hall effect in *gold* and *bismuth*. The semi-circular piece of bismuth, *Bi*, Fig. 22, was fastened to a semi-circular ringplate of copper, *Cu*. The primary current  $I$  entered and left by large electrodes *A* and *B*, so that the stream lines of the current in the plate were radial. From the Hall electrodes *a* and *b* small wires led off. If the distances of the electrodes *a* and *b* from the centre were small compared with the radius of the large circle

and large in comparison with the radius of the electrode  $A$ , then, as Boltzmann <sup>(85)</sup> has pointed out, the formula for the Hall coefficient  $R$  will be the same as that for the usual rectangular plate.

In the case of the circular plate, Fig. 23, the bismuth,  $Bi$ , was cut radially from  $A$  to  $D$ , leaving a narrow slot. A complete copper ringplate surrounded the  $Bi$  plate, the primary electrodes were  $A$  and  $B$ , and the Hall electrodes were  $a$  and  $b$ . If the latter electrodes are located with respect to  $A$  as noted in the case of the semi-circular plate, the usual formula for  $R$  holds for this type of plate also.

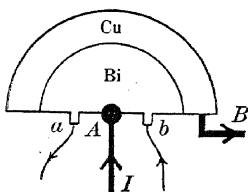


Fig. 22.

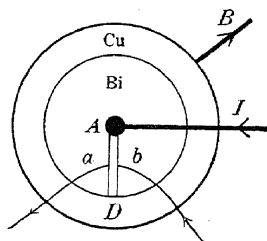


Fig. 23.

It would seem that the only advantage of circular plates is that a more uniform and symmetrical field may thereby be secured throughout the metal examined. Van Everdingen <sup>(175)</sup>, 1896, found that the Hall effect in *bismuth*, under the same conditions, was smaller in round than in rectangular plates.

## 6. Condenser Discharge Method

Ettingshausen and Nernst <sup>(174)</sup>, 1886, measured the Hall effect in *bismuth* and *tellurium* by sending the discharge from a condenser through the plate while it was in a magnetic field. The condenser was charged by 100 cells of a Planté battery of 200 volts. The Hall current was measured by means of a ballistic galvanometer. The results agreed well with those found by the direct current method.

Moretto <sup>(378)</sup>, 1900, also employed the condenser method

in determining the Hall effect in *bismuth*. The Hall current was measured by the Cardani alkalimetric method, p. 23. Since the discharge current was damped by the resistance in the voltameter cells, the current was probably not oscillatory. The Hall current seemed independent of the capacity of the condenser and the length of the spark gap.

### 7. Lebre's Method

In the method devised by Lebre<sup>(314)</sup>, 1895, for measuring the Hall effect, the current from the secondary electrodes, *C, D*, is led through one set of coils in a differential galvanometer, *G*, Fig. 24, while the primary current *I* is shunted at

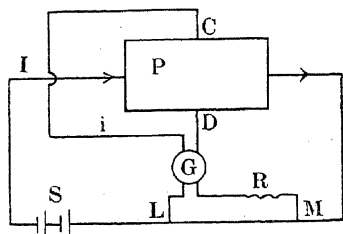


Fig. 24.

*L* and *M*, and a portion of the main current is sent through another set of coils in the galvanometer in the opposite direction. By means of an adjustable resistance *R*, the shunted current is made to counteract the Hall current, so that the deflection

of the galvanometer is zero. The above method was modified by having a "by-current" wire with variable resistance, leading from some point outside of *L—M* on the primary current to the galvanometer, in order to counteract the initial difference of potential at the secondary electrodes.

Lebre<sup>t</sup> claimed that his method, since the primary current is only closed for an instant, eliminated errors due to thermoelectric currents set up by the heating effects of the primary current in other methods.

Van Everdingen<sup>(175 et seq.)</sup> used Lebre's first method in his extended work on the Hall effect, but pointed out that the second "by-current" method was not altogether accurate, since it introduced an error due to the change in resistance of the plate when the magnetic field was thrown on.

## 8. Moretto's Method

The method employed by Moretto<sup>(378)</sup>, 1900, differs from Righi's only in that the divided primary current,  $I$ , after leaving the plate, passed through electrolytic cells, or voltmeters,  $V_1, V_2$ , Fig. 25, and the strengths of the two partial currents  $I_1, I_2$ , were measured by an alkalimetric process.

When the plate was in the magnetic field, the partial current  $I_1$  would be increased and  $I_2$  diminished, or *vice versa*, and in consequence there would be more electrolytic action in one voltmeter than in the other, and this difference would be proportional to the Hall effect.

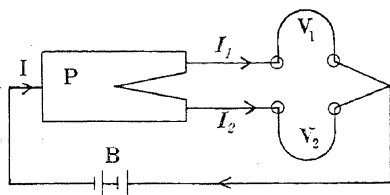


Fig. 25.

The alkalimetric method of measuring the currents  $I_1$  and  $I_2$ , used by Moretto, is due to Cardani<sup>(110)</sup>.

This method of measuring the Hall effect is probably not a very accurate one, for the reason that the error in determining the current through each voltmeter may be very appreciable, and since the Hall effect depends on the difference of these two currents, the magnitude of this effect as computed may be in error by many per cent.

## 9. Barlow's Method

When a metal plate, carrying an electric and a heat current, is placed in a transverse magnetic field, a difference of potential,  $E$ , is set up at the secondary electrodes,  $A$  and  $B$ , Fig. 26, due in part to the Hall effect,  $R$ , and in part to the Nernst effect,  $Q$  (p. 212), and we have

$$E = \frac{RHI}{d} + QHb \frac{dt}{dl},$$

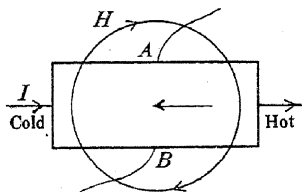


Fig. 26.

in which  $\frac{dt}{dl}$  is the temperature gradient in the plate. Barlow<sup>(44)</sup>, 1903, in his method, kept the ends of the plate at a constant difference of temperature, and sent an electric current in such a direction and of such a strength that the Hall and Nernst potentials were opposite and equal. Then  $E = 0$ , and

$$0 = \frac{RHI}{d} + QHb \frac{dt}{dl}.$$

Hence  $\frac{Q}{R} = -\frac{I}{b \times d} \div \frac{dt}{dl} = -i \div \frac{dt}{dl}$ , where  $i = \frac{I}{b \times d}$  is the current density. The quantities  $i$  and  $\frac{dt}{dl}$  can be measured directly, and  $\frac{Q}{R}$  can be found from these values.

Barlow found the following results for *antimony* and *bismuth*. In the case of antimony, the electric and heat currents flowed through the plate in opposite directions, since in antimony the Hall and Nernst effects have the same signs. In bismuth the two currents were in the same direction.

	$H$	$\frac{dt}{dl}$	$i = \frac{I}{bd}$	Temperature	$\frac{Q}{R}$
Antimony	9130	10° C.	0.82	44° C.	+ 0.0817
Bismuth	9030	12.3	0.99	50	— 0.081
„	9030	18.5	1.57	42	— 0.085

If either  $Q$  or  $R$  is known, the other coefficient may be found.

#### 10. Koláček's Method

The method adopted by Koláček<sup>(294)</sup>, 1904, for measuring the Hall effect, was as follows: The primary current  $I$  entered and left the plate through heavy copper wires  $C, C$ , Fig. 27. One Hall electrode led off from  $A$ , the middle of the upper edge of the plate, while two secondary electrodes  $B$  and  $D$ , were placed equidistant from  $M$ , the middle of the lower edge of the plate. The wires from  $B$  and  $D$  were connected to a rheostat,  $S$ , which was so adjusted that no current flowed

through the galvanometer  $G$ , when the magnetic field was not on.

Novák and Macků<sup>(392)</sup>, 1908, showed that the Hall current decreased as the distance between the wires at  $B$  and  $D$  increased, and that when the wires were placed at the lower

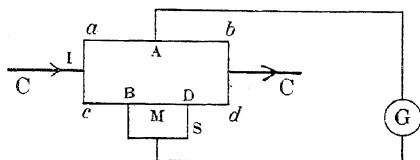


Fig. 27.

corners  $c$  and  $d$ , the effect sank to one half. Raus<sup>(425)</sup>, 1910, confirmed this fact, that the Hall current depended on the distance between  $B$  and  $D$ . Later Kolářček (l. c.) showed from theoretical considerations that, if the primary electrodes are *point* ones and are fastened to the corners  $a$  and  $b$ , the Hall

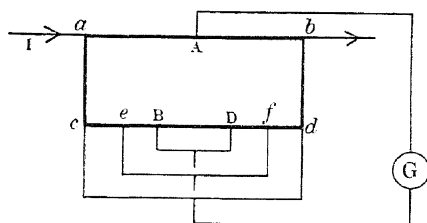


Fig. 28.

effect will be independent of the distance between  $B$  and  $D$ . Raus (l. c.) placed fine bismuth wires along the edges of the plate at  $c, e, B, D, f, d, A$ , Fig. 28, and the primary electrodes at  $a$  and  $b$ . He found that the Hall effect was the same, whether wires  $B$  and  $D$ ,  $e$  and  $f$ , or  $c$  and  $d$  were joined together. Thus Kolářček's theory was confirmed.

### 11. Potentiometer Method

The potentiometer method used by many investigators is outlined in simple form in Fig. 29. The difference of



potential set up by the magnetic field at the Hall electrodes *A* and *B* is compensated by the E. M. F. of a standard cell *S*, so that no current flows through the galvanometer *G*. The compensating E. M. F. is read from the potentiometer, not shown in detail in the diagram above. The potentiometer null-

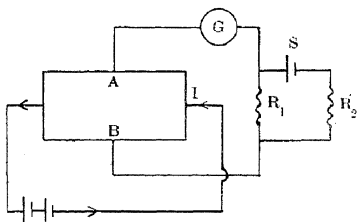


Fig. 29.

method has much to recommend it. Since no current flows from the electrodes *A*, *B*, no Peltier effect is set up at these points, and the thermoelectric current set up by the Peltier difference of temperature is therefore

eliminated. Also the stream lines of the primary current are not distorted by the wandering of a secondary current through the plate.

This method also eliminates error due to uncertainty of the resistance of the secondary circuit.

Of course this method does not eliminate the error due to the superposition of the Ettingshausen effect on the Hall effect. The former effect must be determined separately, see p. 146, and subtracted from the sum of the two effects.

## 12. Alternating Current Method

Des Coudres<sup>(133)</sup>, 1900, determined the Hall effect in *bismuth* by means of an alternating current, *I*, Fig. 30, which traversed at the same time both the metal plate and the

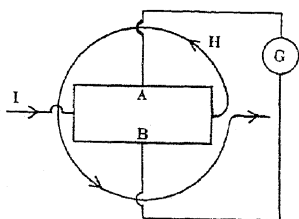


Fig. 30.

electromagnet. Thus the reversal of the current in the plate and in the electromagnet were simultaneous. Consequently the difference in potential at the Hall electrodes, *A*, *B*, did not change sign, and a direct current flowed through the galvanometer *G*. In order to

keep the current in the plate and in the magnet in the same phase no iron was used in the circuit. Hence only low magnetic fields could be obtained. Des Coudres developed the following expression for the Hall effect determined by the alternating current method:

$$R = \frac{Ed}{CI^2},$$

in which  $d$  is the thickness of the plate,  $E$  the difference of potential at the secondary electrodes,  $I$  the primary current through the plate, and  $C$  the strength of the field when 10 amperes pass through the coils of the electromagnet. The above expression is independent of the frequency and form of the curve of the primary current. In the table below are given the Hall effects as determined by Des Coudres with a frequency of 50 per sec.

*Hall Effect with Alternating Current*

Metal	$C$	$R$	Metal	$C$	$R$
Bismuth . .	27.4	— 4.6	Tin . . . .	27.4	— 0.000013
Copper . .	—	— 0.0007	Mercury .	—	— 0.00052
Aluminium	—	— 0.00019	Bismuth Amalgam }	—	— 0.00124

Von Traubenberg<sup>(548)</sup>, in 1905, determined the Hall effect in bismuth with an alternating current of 60 cycles per sec. He found that the effect agreed in magnitude with that determined by other methods.

Zahn<sup>(612)</sup>, in 1911, pointed out that Des Coudres' method was not suited for high frequency currents, since such currents set up induced currents in the plate and bring about differences of temperature at the Hall electrodes, and in consequence thermoelectric currents are superposed on the Hall current. Zahn found that the Hall effect in *bismuth* and *antimony* determined by the Des Coudres method was smaller than that found by the direct current method. This difference he attributed to the fact that the former method eliminated the

temperature difference at the secondary electrodes, that is, the Ettingshausen effect, p. 146, which in the direct current method is superposed on the Hall effect. In the Des Coudres method, when the frequency is not too high, the secondary electrodes *A* and *B*, Fig. 30, are equally heated, and an *isothermal* Hall effect is the result.

Zahn's results were in accord with Gans' <sup>(208)</sup> theory, p. 94.

Smith <sup>(489)</sup>, in 1912, in commenting on the method of Des Coudres said: "If the Hall effect is caused by the deflection of the electrons in the metal in some such way as the cathode rays are deflected by a transverse magnetic field, the electrons in the plate would always be deflected in the same way, since the magnetic field and the current in the plate alternate simultaneously. If for this or any other reason

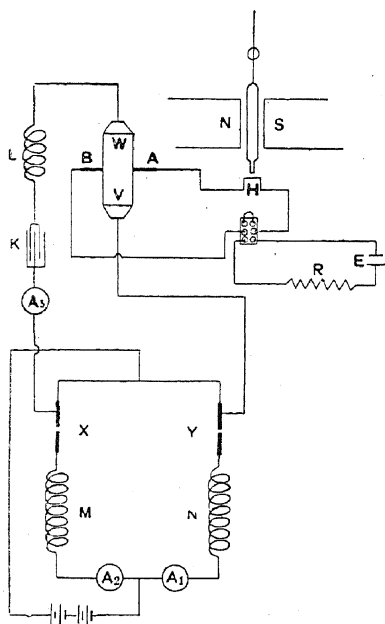


Fig. 31.

an appreciable time is required to build up the Hall electromotive force at the electrodes, the method of Des Coudres is not adapted to detect this fact."

To avoid this objection, Smith modified the method by having the direction of the magnetic field remain constant, while the current through the plate alternated. The Hall E.M.F. therefore was an alternating one.

In Fig. 31 there is an outline of the apparatus used by Smith. The alternating currents were obtained from two Duddell arcs, *X* and *Y*, in parallel. The *bismuth* plate, *W*—*V*,

was placed in the secondary circuit, that contained a self-induction coil,  $L$ , and a condenser,  $K$ . The high frequency currents were measured by a hot wire ammeter  $A_3$ . The frequency of the current was varied by changing  $L$  and  $K$ . The Hall current was measured by a Duddell thermal galvanometer. The table below gives the results of Smith's determinations in a field of 12,500 gauss.

*Hall Effect in Bismuth at High Frequencies*

Plate	Method	Current	Frequency	$R$
A	Potentiometer	3.58 amp.	0	-2.77
"	Duddell Galv.	3.58	0	2.75
"	"	3.60	60	2.75
"	"	3.45	30,000	2.73 (2.73)
"	"	4.00	50,000	2.74 (2.70)
"	"	3.65	120,000	2.74 (2.59)
B	Potentiometer	3.50	0	2.47
"	Duddell Galv.	3.50	0	2.46
"	"	3.48	60	2.46
"	"	3.60	30,000	2.47 (2.46)
"	"	3.80	50,000	2.50 (2.47)
"	"	3.60	120,000	2.47 (2.33)

The above results show that the Hall effect is independent of the frequency of the alternating current through the plate, and that the magnitude of the effect is the same for the alternating and direct current methods.

### 13. Hall Effect in a Circular Plate

Volterra<sup>(566)</sup>, 1915, showed from theoretical considerations, that if a current of electricity,  $I$ , Fig. 32, enters a circular metal plate by a point electrode at  $A$ , and leaves the plate by a point electrode  $B$ , each electrode being situated at the perimeter of the plate, then the difference of potential set up by a magnetic field between the equipotential points  $C$  and  $E$  is the same as that set up between the equipotential points  $D$  and  $F$ , and also the difference in potential set up by the

field between  $D$  and  $E$  and between  $C$  and  $F$  will be equal. The points  $E$  and  $F$ , and  $C$  and  $D$  are on stream lines of the primary current, and  $E$  and  $C$ , and  $D$  and  $F$  are on equipotential lines.

Alimenti<sup>(20)</sup>, in 1915 and 1916, tested experimentally the validity of Volterra's theorem of the four vertices, or principle of reciprocity.

The primary current entered and left the circular plate of *bismuth* by point electrodes at  $A$  and  $B$ , Fig. 33, situated at the middle points of two radii perpendicular to each other.

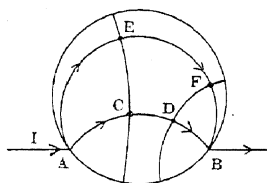


Fig. 32.

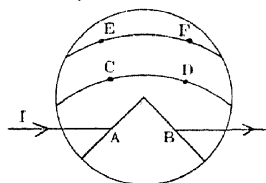


Fig. 33.

The points  $C$ ,  $D$ , and  $E$ ,  $F$ , are respectively two generic points of two curves that belong to the reciprocal points of  $A$  and  $B$ .

The table below gives the results of the measurements of the difference in potential between the points  $C$ ,  $D$ ,  $E$ ,  $F$ , taken two and two. This difference is represented by  $\delta$  when the magnetic field is on in one direction, and by  $\delta'$  when the field is reversed. The Hall effect is given in terms of  $\frac{\delta - \delta'}{2}$ .

	Field +	Field -	$\delta$	$\delta'$	$\frac{\delta - \delta'}{2}$
$C, D$	574.116	574.116	—	—	0
$E, F$	560.307	560.307	—	—	0
$D, E$	557.899	557.803	342	246	48
$C, F$	559.994	559.898	435	339	48
$C, E$	553.326	551.443	227	108	59.5
$D, F$	566.450	566.569	116	—3	59.5

The difference of potential between  $D$  and  $E$  is the same as that between  $C$  and  $F$ , and the difference between  $C$  and  $E$  and between  $D$  and  $F$  is the same. Thus Volterra's theorem of the four vertices, or the principle of reciprocity, is confirmed.

Bordonaro <sup>(90)</sup>, 1915, also confirmed experimentally this principle of reciprocity. He pointed out the necessity of having all point-electrodes. When strip or clamp electrodes are used, internal currents flow round in the metal, and the superposition of these currents on the original current increases the energy consumed by the Joule effect, and a *global* resistance is the result, and the true value of the Hall effect cannot be measured.

#### 14. Errors in Determining the Hall Effect

1. *Length of Plate.* If the metal plate is short and the metal clamps at the ends of the plate are within the magnetic field, the Hall effect becomes complicated.

Ettingshausen and Nernst <sup>(174)</sup> in 1886 investigated the influence of the length of the plate upon the magnitude of the Hall effect in *bismuth*. They found the effect to be less in short plates. The table below gives the results for various lengths of the plate, which was originally 7 cm. long and 1.14 cm. wide. The plate was kept the same width, and was cut to the indicated ratios of length to width,  $l:b$ .

*Hall Effect in Bismuth*

$l:b$	6:1	3:2	1:1
$R$	—6.29	—6.28	—5.61

These investigators advocated the use of plates in which the ratios  $l:b$  is at least 3:2, and better 2:1.

Hall in 1888 <sup>(243)</sup> measured the Hall effect in two strips of *steel*, one about 8 cm. long and 2 cm. wide, the other 1.1 cm. long and 2 cm. wide. The effect in the short strip was about 20% less than in the longer one.

The smaller Hall effect in the short plate is due to the fact that all the Hall current does not flow through the secondary electrodes, a portion of the current leaking back through the plate to the primary clamps.

In a very short plate an appreciable temperature gradient may be set up in consequence of the Peltier effect at the junctions of the plate and the primary lead wires or clamps. In this case the Hall effect becomes complicated with the thermomagnetic effects.

2. *Thermoelectric Currents.* The magnetic field sets up in the plate a difference of temperature at the secondary electrodes, the Ettingshausen effect, p. 146. If, as is usually the case, the secondary lead wires are of a metal different from that of the plate, a thermoelectromotive force will be superposed on the Hall difference of potential. In all Hall effects determined prior to the discovery of the Ettingshausen effect in 1880, as well as in some later determinations, no allowance for this latter effect was made. However, the E. M. F. arising from the Ettingshausen effect is usually small compared with the Hall effect, so that the error due to ignoring the first effect is small. The Ettingshausen effect may be largely eliminated as an influence on the Hall effect by using secondary lead wires of the same material as the plate. However, there will always be a slight difference in the thermoelectric properties of the wire and plate, and hence a small thermoelectric current will be superposed on the Hall current. The bath method, and the alternating current method eliminate in whole or in part the error due to the Ettingshausen effect. The better way is to measure this latter effect direct, and then correct the Hall effect for it.

3. *Location of Hall Electrodes.* If the secondary electrodes are not at equipotential points, that is, if one electrode is in advance of the other, small longitudinal effects will be introduced. Errors due to this condition are eliminated by reversal of the magnetic field, since the Hall

effect is reversed with the field, while the longitudinal effects are not.

4. *Nature of Hall Electrodes.* La Rosa and De Luca <sup>(459)</sup>, 1918, measured the Hall effect in a thin plate of *platinum* having in turn secondary electrodes of *bismuth*, *copper*, *platinum*, and *antimony*. The magnitude and sign of the effect varied with the nature of the electrodes. The change from the normal effect was too great to be accounted for by the thermoelectric current set up by the Ettingshausen difference of temperature at the edges of the plate.

The authors attributed this change in the Hall effect to a new difference of electron concentration at the two junctions of the secondary electrodes with the plate, set up by the magnetic field.

When the junctions of the electrodes are at the same temperature and no field is on, there will be a difference of electronic concentration and pressure at the two junctions, but the two differences in pressure will be equal and in opposite direction, and consequently no current flows. When the magnetic field is thrown on, an unequal difference in electron concentration is set up, and in consequence a current is superposed on the Hall current.

5. *Location of the Plate.* In the case of magnetic metals, an error may be introduced by strains produced in the plate by the magnetic field. This error may be eliminated by placing the plate snugly against one plane-faced pole of the electromagnet. The pole of course should be covered with insulating material.

6. *Thickness of the Plate.* The inability to determine accurately the thickness of a plate, especially if very thin, introduces an error. The thickness of a plate that is not perfectly smooth cannot be measured accurately by means of calipers. The determination of the thickness by the method of weighing is not always accurate, since the density used may be an assumed one.



In the case of thin metallic films, the method of finding the thickness from the electrical resistance is open to error, since the resistance of thin films, as pointed out by Ettingshausen<sup>(167)</sup>, Longden<sup>(337)</sup>, Sir J. J. Thomson<sup>(521)</sup>, and others, does not follow the law that holds for thick portions of metal.

7. *Impurities in the Plate.* The presence in the metal of impurities affects very materially the magnitude of the Hall effect. The discrepancies in the results of different investigators is doubtless often due to varying quantities of impurities in the metal studied.

8. *Other Sources of Error.* In addition to errors due to observation and to mechanical causes, thermal currents due to inherent lack of uniformity of temperature in the plate and electrodes, change in property of metal due to soldering, induced currents set up in the secondary circuit by a changing magnetic field, changes of resistance and temperature and thermoelectric height in the magnetic field may introduce errors in the measurements of the Hall effect.

*Note.* The several sources of error above mentioned affect more or less the determination of all the effects herein treated.

### CHAPTER III

#### VARIATIONS OF THE HALL EFFECT

##### 1. Variation of the Hall Effect with Current Density

In general the Hall electromotive force varies directly as the primary current and inversely as the thickness of the plate, and is independent of the cross-section area of the plate.

Hall<sup>(243)</sup>, 1888, observed that the Hall coefficient was independent of the area of cross-section of the plate in the case of *non-magnetic metals*, while in the *ferromagnetic metals*, the coefficient depended greatly upon the cross-sectional area, and was greater in thicker specimens. This is due to

the fact that the magnetic induction, to which the effect is proportional, is greater in a plate whose thickness is large compared with its width.

Moretto <sup>(378)</sup>, 1900, investigated the dependence of the Hall coefficient on the current density in *gold* and *bismuth*. The gold sheet used was 0.0000088 mm. thick. It was found that for currents greater than 0.4 ampere, the Hall coefficient was constant, while for currents less than 0.4 ampere, the coefficient increased. The effect was measured by the Cardani alkalimetric method. Since this method is probably not very reliable, some doubt may rest on Moretto's conclusions.

Moreau <sup>(375)</sup>, 1901, measured the Hall effect in very thin films of *silver* deposited by electrolysis on glass, and in *nickel* deposited on silver films. The table below gives the results for silver.

*Hall Effect in Films of Silver*

Thickness	$R \times 10^4$	Thickness	$R \times 10^4$
$130 \times 10^{-6}$ mm.	-5.78	$67 \times 10^{-6}$ mm.	-7.13
121	5.90	61	7.97
105	6.07	49	9.25
86	6.37	48	11.19
76	6.85	44	12.57

The above table shows that as the thickness of the film decreases the Hall coefficient increases. For thicknesses down to 50 millionths mm., Moreau proposed the following formulae:

$$\text{For silver, } R = \frac{-d}{2160(d - 25 \times 10^{-7})};$$

$$\text{For nickel, } R = \frac{-d}{30(d - 23 \times 10^{-7})};$$

in which  $d$  is the thickness of the film in cm. In each case  $R$  increases as  $d$  decreases.

For thicknesses below 0.000044 mm., the Hall coefficient diminished rapidly.

Instead of the usual formula for the Hall effect, Moreau proposed the expression

$$E = \frac{RHI}{d-d_1},$$

where  $d$  is the thickness of the plate, and  $d_1$  the critical thickness. For silver,  $d_1 = 25 \times 10^{-7}$  cm., and for nickel  $d_1 = 23 \times 10^{-7}$  cm.

Moreau explained the variation of  $R$  with the thickness of the plate as due to the influence of transition layers of silver and glass, and silver and air, at the boundaries of the plate. Vincent<sup>(557)</sup>, 1900, measured the resistance of films of silver between 0 and 170  $\mu\mu$ , and found that the resistance increased as the thickness decreased, in a manner similar to the variation of the Hall effect in very thin films. Moreau also measured the resistance of thin films and found that  $\frac{R}{\rho}$  was constant. Since  $\tan \Phi = \frac{RH}{\rho}$ , and  $\frac{R}{\rho}$  is constant, the angle of rotation of the equipotential lines is independent of the thickness of the film, so long as  $H$  is constant.

Patterson<sup>(404)</sup>, 1901, confirmed the results of Moreau.

Sir J. J. Thomson<sup>(521)</sup>, 1901, showed from theoretical considerations that the specific resistance of thin films should be a function of the thickness.

McKay<sup>(351)</sup>, 1904, studied the relation of the Hall effect to the current density in *gold* leaf, and came to the conclusion that there was an apparent slight increase in the coefficient. This, he thought, may have been due to experimental error, or to stresses set up in the gold leaf by the electric current, which heated the plate and caused the paraffin and cement about the gold to expand.

In 1921, Stewart<sup>(507)</sup> and Wait<sup>(569)</sup> claimed that the Hall coefficient was independent of the current density in *silver* films chemically deposited, although the specific resistance of the films was several hundred times that of bulk silver.

## 2. Variation of the Hall Effect with the Magnetic Field

In general, for weak fields, the Hall effect is proportional to the strength of the field, while for strong fields, the proportionality does not always hold, especially in the case of the ferromagnetic metals, and in the highly crystalline metals bismuth and antimony. R.H

Hall <sup>(237)</sup>, 1881, found that the effect in *nickel* began to fall off when a field of about 5000 units had been reached. In higher fields the effect seemed to reach a saturation value, as shown in Fig. 34, in which  $R \propto H$  is plotted on an arbitrary scale.

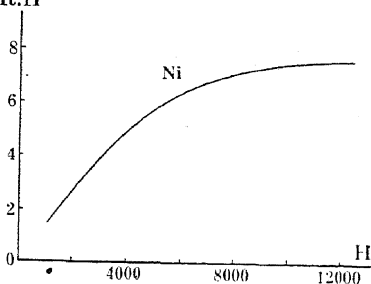


Fig. 34.

Ettingshausen and Nernst <sup>(173)</sup>, 1887, examined the change in the Hall effect in *bismuth* with the magnetic field. The results of their investigation are given in the table below.

Hall Effect in Bismuth

H	1650	2520	3640	6080	8170	9830	11100
R	-10.27	-9.50	-8.72	-7.14	-6.12	-5.40	-4.95

From the above it is seen that the Hall effect coefficient in bismuth falls off steadily as the field increases.

The same investigators found that, in *bismuth alloyed* with 1.46% of *tin*, the Hall coefficient decreased as the field increased, and that between 6000 and 7000 gauss the effect became zero and then changed sign.

The Hall effect in *gold* and *silver* was found by Kundt <sup>(303)</sup>, 1893, to be directly proportional

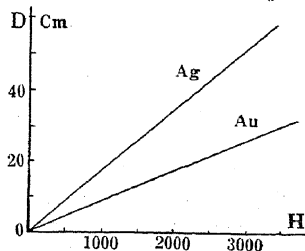


Fig. 35.

to the field strength, as shown in Fig. 35. The deflection  $D$  of the galvanometer, which is proportional to the Hall effect, is plotted against the field strength,  $H$ .

The variation of the Hall effect in *iron*, *nickel* and *cobalt* with field strength was also determined by Kundt. The curves in Fig. 36 exhibit this variation.

The effect in each metal approached a saturation value in higher fields. The effect varied as the magnetization and not as the field strength.

To Kundt is usually given the credit of first pointing out, in 1893, that the Hall effect in ferromagnetic metals

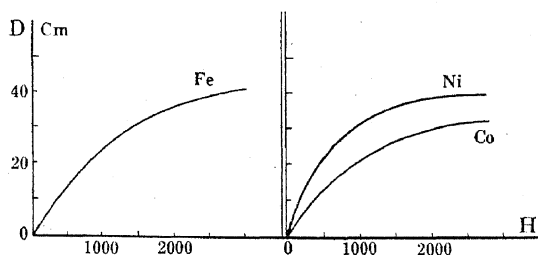


Fig. 36.

should vary as the intensity of magnetization and not as the strength of the magnetic field. Kundt<sup>(302-3)</sup> allowed polarized light to pass through thin transparent films of *iron*, *nickel* and *cobalt*, and found that the rotation of the plane of polarization in the magnetic field and the Hall effect, in fields up to 22,000 gauss, were proportional to each other. He assumed that the rotation of the polarized light was proportional to the magnetization, and then argued that the Hall effect likewise should be proportional to the magnetization.

Hall<sup>(243)</sup>, however, had already in 1888, in a paper on the Hall effect in iron, cobalt and nickel, written: "When a piece of *iron*, *cobalt*, or *nickel* is made to approach the state of 'magnetic saturation', the transverse current [Hall effect] obtained from it increases somewhat less rapidly than the magnetic induction through the metal, but experiments with

very highly magnetized iron and nickel indicate that this transverse current tends towards a fixed limit rather than a maximum followed by a decline."

Van Everdingen<sup>(183)</sup>, 1899, measured the Hall effect in *bismuth* through a considerable range of field strength and of temperature. The curves in Fig. 37 show that at low temperatures the effect decreases in higher fields more rapidly than at higher temperatures.

Baker<sup>(40)</sup>, 1902, found that the Hall coefficient for *gold* was constant for fields from 12 up to 21,000 gauss.

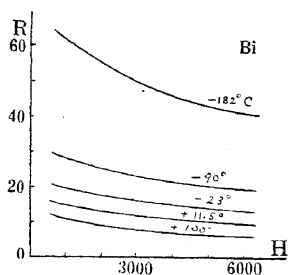


Fig. 37.

Lownds<sup>(34+5)</sup>, 1903, measured the Hall effect, at various temperatures and field strengths, in a plate of *bismuth* cut from a *crystal* so that the main crystallographic axis was parallel to the length of the plate.

The table below shows that the Hall effect decreases as the field increases, but contrary to Van Everdingen's results, the effect is greater at high than at low temperatures, and at  $-186^{\circ}\text{C}$ . the effect has a positive sign as shown below.

Hall Effect in Bismuth

Temperature	Primary Current $\parallel$ to Crystal Axis			Primary Current $\perp$ to Crystal Axis		
	$16^{\circ}\text{C}$ .	$-79^{\circ}\text{C}$ .	$-186^{\circ}\text{C}$ .	$16^{\circ}\text{C}$ .	$-79^{\circ}\text{C}$ .	$-186^{\circ}\text{C}$ .
Field	$R$	$R$	$R$	$R$	$R$	$R$
2120	-11.8	-9.60	+10.4	-10.4	-7.85	+8.89
3120	-11.3	-6.92	+9.13	-10.1	-5.61	+7.62
3500	-11.4	-5.92	+8.85	-9.97	-4.81	+7.51
4980	-10.3	-3.16	+7.88	-9.02	-2.38	+6.51

In all cases in the above table the Hall effect is larger when the primary current is parallel to the principal axis of the crystal.

The following tables give the variation of the Hall effect in *antimony* and *bismuth* with the change in magnetic field, as determined at  $21.5^{\circ}$  C. by Barlow <sup>(44)</sup>, 1903.

*Hall Effect in Antimony*

Field	$R$	Field	$R$
140	+ 0.26 (?)	8190	+ 0.201
720	0.21	10200	0.199
1750	0.21	19200	0.189
4310	0.208	24700	0.186

*Hall Effect in Bismuth*

Field	$R$	Field	$R$
2000	- 3.8	8000	- 2.7
4000	3.3	10000	2.4
6000	3.0	11000	2.3

In each metal the effect falls off with increase of field.

Zahn <sup>(609)</sup>, 1904, observed that in fields of over 8000 gauss there was a slight falling off in the Hall coefficient for *antimony*.

The Hall coefficient for *iron* was found by Barlow <sup>(44)</sup> to fall off at higher fields and to reach a saturation value in a field of 25,000 gauss, at  $22^{\circ}$  C.

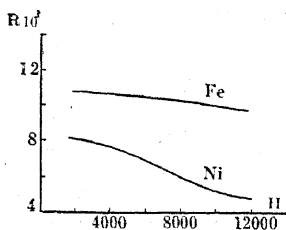


Fig. 38.

On the other hand Zahn <sup>(609)</sup> found that the Hall coefficient for *iron* and *nickel* decreased in fields above 4000 gauss, as shown by the curves in Fig. 38.

The Hall effect in *iron*, *nickel* and *cobalt* was determined by Smith <sup>(486)</sup>, 1910, for various fields and temperatures. The curves, in the Figs. 39 and 40, show that in general the Hall E.M.F. in *iron* and *nickel* is proportional to the field strength, until the maximum intensity of magnetization is approached, which is different for different

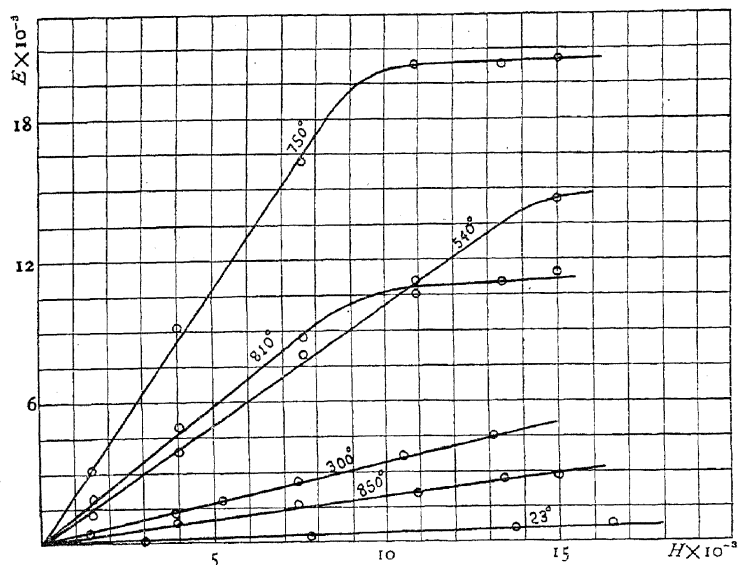


Fig. 39. Electrolytic iron.

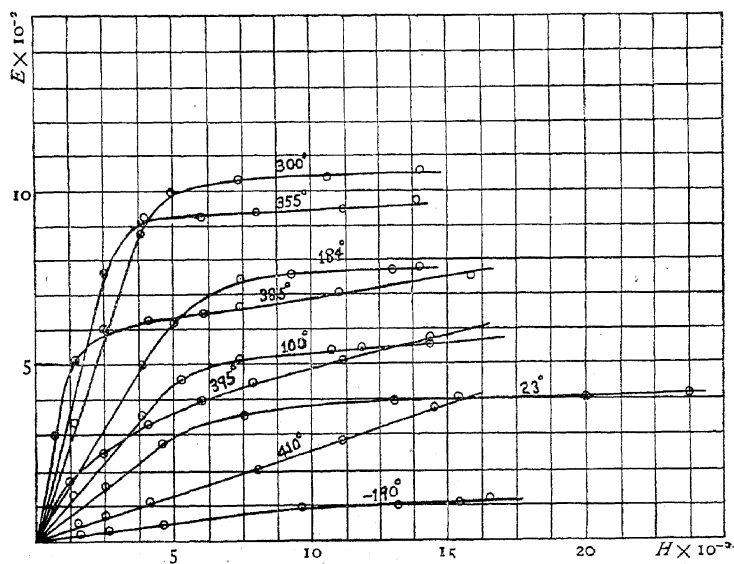


Fig. 40. Nickel.



temperatures. After magnetic saturation, the values of the E. M. F. seem to become constant.

For *cobalt* the Hall E. M. F. varied very much as was the case for iron and nickel.

### *Summary.*

In general, the Hall coefficient in *bismuth* and *antimony* decreases in higher fields and seems to approach a limiting value.

In the *ferromagnetic metals*, the Hall coefficient increases rapidly at first, and then approaches a saturation value in higher fields.

In *all other metals* thus far examined the coefficient is practically independent of the field.

### 3. Variation of the Hall Effect with the Direction of the Magnetic Field:

#### Dissymmetry

Lebret<sup>(314)</sup>, 1895, Van Aubel<sup>(25)</sup>, 1895, Van Everdingen<sup>(176-7)</sup>, 1901, and others observed that the Hall effect in *bismuth* changed its magnitude when the magnetic field was reversed. Lebret employed a circular plate of bismuth to which he attached four electrodes. Turning the plate in its own plane, he found one position in which the dissymmetry disappeared. From a consideration of Goldhammer's<sup>(214)</sup> equations for the Hall effect, Lebret showed that, if the primary electrodes are fastened to the plate in the direction of the principal axis, all dissymmetry should disappear. This was found to be true.

The dissymmetry in the Hall effect appears to be intimately connected with the fact that the resistance of bismuth in a magnetic field is different in two directions perpendicular to each other. This dissymmetry also seems dependent on the crystalline structure of the metal. The plates used by Van Everdingen were made from cast bismuth that had cooled very slowly, and possessed a very uniform crystalline structure. In these plates the dissymmetry was very pronounced, while in electrolytic bismuth, the dissymmetry was very small.

Lebret found that the dissymmetry in weak fields increased a little more slowly than the square of the field, and for stronger fields the dissymmetry increased faster than the first power of the field. Van Everdingen noted that the dissymmetry decreased with the rise of temperature, and more rapidly than the Hall effect.

In *bismuth sulphide* Van Aubel <sup>(29)</sup>, 1903, found that the dissymmetry in the Hall effect was fairly large and varied with the position of the plate in the magnetic field.

The dissymmetry in the Hall effect in *bismuth* was determined by Becquerel <sup>(57)</sup>, 1912, at 20° C., in fields up to about 9000 gauss. Curve I, Fig. 41, gives the variation of the effect when the field is in one direction, while Curve II represents the effect when the field is reversed. There is a decided dissymmetry in the curves. One has changed sign at about 6000 gauss, while the other apparently would change sign above 10,000 gauss.

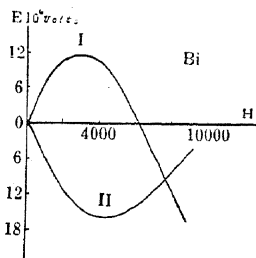


Fig. 41.

Van Everdingen <sup>(176)</sup>, 1901, observed a small dissymmetry in the Hall effect in *antimony*. In *tellurium*, Wold <sup>(604)</sup>, 1916, found a dissymmetry which was proportional to the square of the field strength, and proportional to the longitudinal change of resistance. This seems to confirm the theory of Lebret and Van Everdingen, that the dissymmetry in the Hall effect is due to a change of resistance in the magnetic field.

It seems probable that a dissymmetry in the Hall effect may be found in any metal that is highly crystalline, and not of the isometric system.

#### 4. Reversal of the Hall Effect in the Magnetic Field

Ettingshausen and Nernst <sup>(173)</sup>, 1887, found that on the addition of 1 to 6% of *tin* to *bismuth* the Hall effect changed sign in a field of 11,000 gauss.

Beattie <sup>(48)</sup>, 1895, discovered that in thin plates of *bismuth* the Hall effect changed from a negative to a positive value in fields from 60 to 100 gauss. In thick bismuth plates the effect did not change sign in any field employed.

Van Everdingen <sup>(184)</sup>, 1900, observed that in one of his plates, cut from a *bismuth crystal*, the Hall effect was negative when the principal axis was perpendicular to the lines of force in fields of 2900 to 5000 gauss, and that the effect had a positive sign when the axis was parallel to the lines of force.

The inversion of the Hall effect in *bismuth* at 20° C. was observed by Becquerel <sup>(57)</sup>, 1912, and is shown in Fig. 41, above. At -190° C. the inversion was found by Becquerel

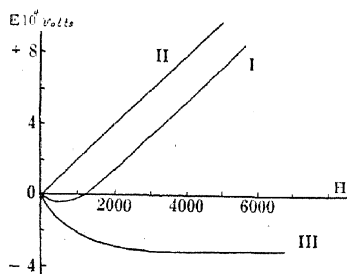


Fig. 42.

to take place in weaker fields, as shown by Curve I in Fig. 42. In higher fields the effect became linear and positive. Becquerel supposed that the observed effect represented by Curve I, was the resultant of two effects, one positive and linear, represented by Curve II, and

one negative, Curve III, which reached a saturation value for fields above 3000 gauss. This saturation effect suggests a molecular orientation under the influence of the magnetic field, while the linear positive component is in accord with the theory that the Hall effect is due, in part at least, to the motion of free electrons under electric and magnetic forces.

Reversals, similar to that observed by Becquerel in *bismuth*, were noted by Smith <sup>(494)</sup>, 1917, in *bismuth-lead alloys* containing 3%, 5%, and 10% of lead. In each case the effect reached a negative maximum, changed sign, and became linear in higher fields.

## 5. Variation of the Hall Effect with Temperature

a. *Ferromagnetic Metals.* Hall <sup>(241)</sup>, 1885, in a summary of an account of the determination of the Hall effect in certain metals, says: "A fall of  $1^{\circ}$  C. in temperature causes in the R. P. (rotational power) of

<i>Iron</i>	a fall of $2/3\%$ approx.
<i>Steel, soft</i>	" " " $1/3\%$ "
" tempered	" " " $1/3\%$ "
<i>Cobalt</i>	" " " $1\%$ "
<i>Nickel</i>	" " " $2/3\%$ " "

These values apply to temperatures from  $0^{\circ}$  to  $30^{\circ}$  C.

The change in the Hall effect was studied over a wider range of temperature by Clough and Hall <sup>(122)</sup>, 1892. The

curves in Fig. 43 show that the effect in *steel*, *S*, seems to fall off slightly beyond  $300^{\circ}$  C., while the effect in *nickel* reaches a maximum at about  $200^{\circ}$  C., and then falls off rapidly and seems to approach zero above  $350^{\circ}$  C. The peculiar behavior of the effect in nickel, the authors attributed

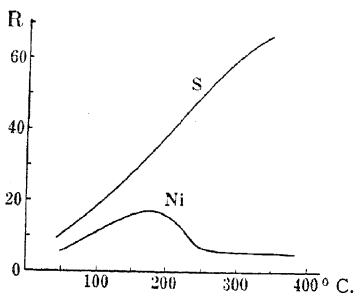


Fig. 43.

to the reversal of the "specific heat of electricity" in the neighborhood of  $200^{\circ}$  C., and to the falling off in the permeability of nickel near  $200^{\circ}$  C. They also called attention to the fact that the temperature-permeability curves of nickel bear a striking resemblance to the Hall effect curve. The Hall coefficient *R* in Fig. 43 is plotted on a relative scale, not absolute.

Zahn and Schmidt <sup>(616)</sup>, 1907, studied the Hall effect in *Heuster magnetic alloys*, and found that the effect changed with temperature, from  $-186^{\circ}$  to  $23^{\circ}$  C., much as it does in the regular ferromagnetic metals.

The curves in Fig. 44 give the change in the Hall effect

with temperature in *iron* and *nickel* as determined by Frey <sup>(206)</sup>, 1908. The effect in iron reaches a maximum at about  $650^{\circ}\text{C}$ ., while in nickel the maximum occurs between  $250^{\circ}$  and  $300^{\circ}\text{C}$ .

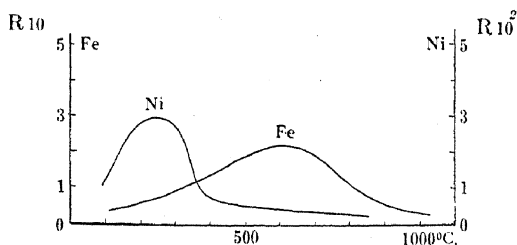


Fig. 44.

The Curie points for iron and nickel are respectively  $775^{\circ}$  and  $360^{\circ}\text{C}$ ., and it would seem that the maximum effect and the Curie point are intimately related.

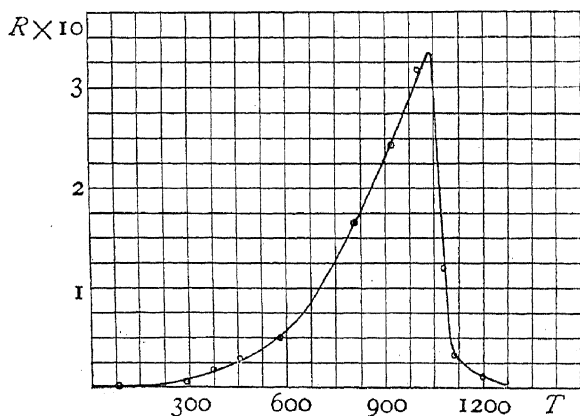


Fig. 45.

Smith <sup>(486)</sup>, 1910, determined the Hall effect in *iron*, *nickel* and *cobalt*, through a wide range of temperature. The curves in Figs. 45 and 46 exhibit maxima somewhat similar to those found by the preceding investigators.

The maximum effects, as seen from the above Figs. 45 and 46, occur at temperatures above those found by Clough

and Hall, and by Frey. This difference may be due to impurities in the metals, or to different physical properties.

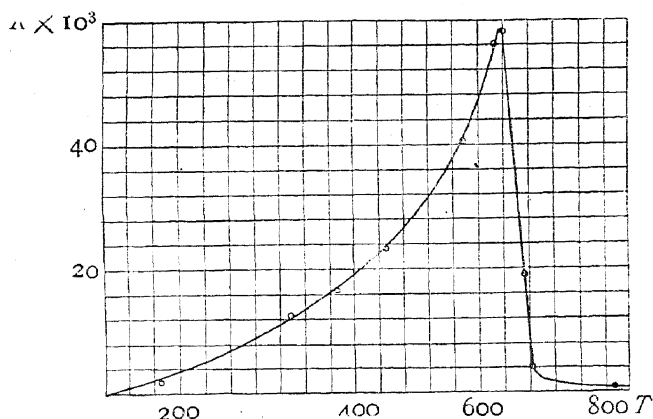


Fig. 46.

of the specimens used. The curve for cobalt was of the same general character as those for iron and nickel.

The Hall effect in *nickel* was determined at very low temperatures by Onnes and Beckman<sup>(356)</sup>, 1912. The curves

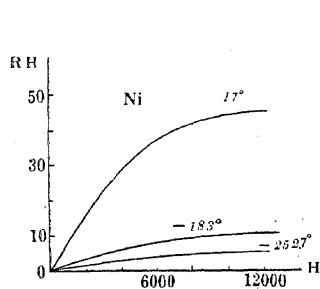


Fig. 47.

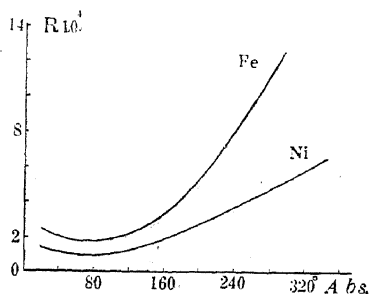


Fig. 48.

in Fig. 47 show that the effect is less at low temperatures, and that at the lower temperatures the effect is more nearly linear. The values of  $R$  are given in the table below.

Alterthum<sup>(12-3)</sup>, 1912, also determined the Hall effect in *iron* and *nickel* at very low temperatures, from  $-253^\circ C.$  to

## Hall Effect in Nickel

290.5° Abs.		90° Abs.		20.3° Abs.		14.5° Abs.	
$H$	$R \times 10^4$	$H$	$R \times 10^4$	$H$	$R \times 10^4$	$H$	$R \times 10^4$
3000	1.21	3000	1.08	2700	1.08	2100	1.00
5000	1.07	5000	0.95	5000	0.98	5000	0.95
7000	0.90	7000	0.80	7000	0.80	7000	0.85
10000	0.70	10000	0.60	10000	0.65	10000	0.65
15000	0.57	15000	0.48	15000	0.50	15000	0.50

18° C. The variation of the effect with temperature differs from the results of others in that the effect first decreases from  $-253^{\circ}$  C. to about  $-200^{\circ}$  C., and then increases, Fig. 48.

## Summary.

In general, the Hall coefficient in the *ferromagnetic metals* increases with temperature to a maximum in the neighborhood of the Curie point, then decreases rapidly, becomes very small at high temperatures, and would seem to approach zero at the melting point of the metal.

b. *Non-Magnetic Metals*. Hall<sup>(247)</sup>, 1885, stated that, between the temperatures  $0^{\circ}$  and  $30^{\circ}$  C., the Hall effect in *copper, zinc, gold, antimony, and bismuth* apparently increased slightly with fall in temperature.

*Bismuth*. Leduc<sup>(325)</sup>, 1884, observed that the Hall effect in *bismuth* fell off about 0.39% per  $1^{\circ}$  C. rise in temperature, and that the effect in *silver* diminished 0.4 to 0.5% per  $1^{\circ}$  C. increase in temperature.

Ettingshausen and Nernst<sup>(174)</sup>, 1887, measured the change in the Hall effect in *bismuth* with change in temperature, with the following result:

Temperature	$0^{\circ}$ C.	$21^{\circ}$ C.	$99^{\circ}$ C.
$R$	-8.1	-7.3	-4.1

Drude and Nernst<sup>(254)</sup>, 1891, obtained the following values for the Hall effect in *bismuth*, at several temperatures. The effect at  $14^{\circ}$  C. is taken as unity.

Temperature	14° C.	100° C.	243° C.
$R$	1.00	1.23	0.23

In this case the effect increased, and then decreased with rise in temperature.

The variation of the Hall effect with temperature in *bismuth*, as determined by Lebre<sup>(344)</sup>, 1895, is shown by the curve in Fig. 49. The coefficient  $R$  is plotted on an arbitrary scale. The effect reaches a maximum at about 0° C., then decreases.

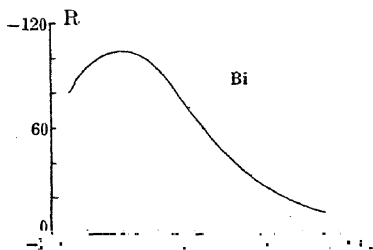


Fig. 49.

The curves in Fig. 50 exhibit the change of the Hall effect with temperature in *bismuth* as measured by Van Everdingen<sup>(183)</sup>, 1900, at temperatures from about 91° Abs. to 373° Abs. These curves show that the effect falls off

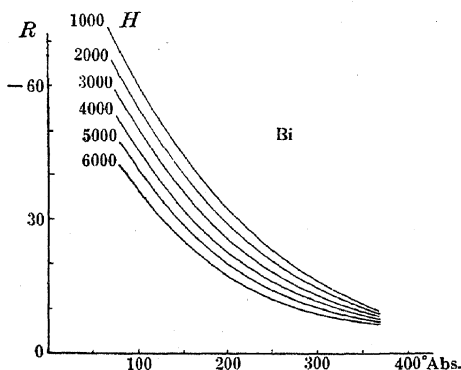


Fig. 50.

at first rapidly with increase in temperature in all fields, and they indicate that the effect approaches zero as the melting point of bismuth is reached.

Lownds<sup>(344)</sup>, 1901, measured the Hall effect in a *bismuth*



*crystal* (cut parallel to its crystallographic axis), at temperatures  $-186^{\circ}\text{C.}$ ,  $-79^{\circ}\text{C.}$ , and  $16^{\circ}\text{C.}$

Curves I and II, Fig. 51, exhibit the change in the effect with temperature, when the primary current was respectively parallel and perpendicular to the crystal axis. The field was 4980 gauss. These curves resemble the lefthand continuation of the Lebet curve, Fig. 49.

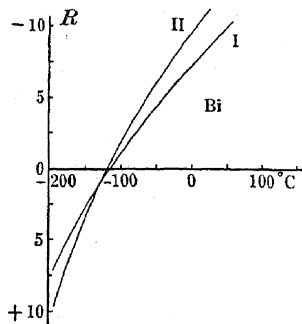


Fig. 51.

The change in the Hall effect in *bismuth* between temperatures  $15^{\circ}$  and the melting point  $270^{\circ}\text{C.}$  was measured by von Traubenberg<sup>(548)</sup>, 1905.

The effect was found to fall toward zero as the melting point was approached.

Onnes and Beckman<sup>(395)</sup>, 1912, made extensive investigations of the variation of the Hall effect in *bismuth* at temperatures of liquid air, liquid hydrogen, and at lower temperatures. The Hall effect,  $R$ , in one plate of compressed electrolytic bismuth, No. III, at  $-252.7^{\circ}\text{C.}$  was found to approach 100, the largest value yet obtained. In the table below it is seen, that at this extremely low temperature the Hall effect increases, while at ordinary temperatures the effect decreases, with field strength.

Hall Effect in Bismuth at  $-252.7^{\circ}\text{C.}$

$H$	2850	4700	6675	8275	10160	11100	12220
$R$	-86.6	-90.7	-93.5	-98.5	-99.2	-99.4	-99.5

In another plate of bismuth, cut from a crystal and placed in the field so that its principal axis was perpendicular to the magnetic lines of force, the above investigators found that the Hall effect became positive at the temperature of liquid hydrogen.

The results are given in the table below. In this plate the effect diminished with increase in field at both temperatures.

*Hall Effect in Bismuth Crystal*

17° C.					
<i>H</i>	2010	3740	5870	8250	10270
<i>R</i>	— 9·95	— 8·18	— 6·58	— 5·11	— 4·31
— 253·7° C.					
<i>H</i>	1850	3700	5800	8700	11080
<i>R</i>	+ 9·72	+ 7·03	+ 5·79	+ 5·02	+ 4·79

These results show the great difference between the behavior of crystal bismuth and electrolytic bismuth in a magnetic field.

Becquerel's idea, p. 44, that the Hall effect is made up of two components, was tested by the above observers. They compared the observed values of  $R \times H$  with those computed from the equation

$$R \times H = aH + b.$$

The plate they used was an electrolytic one. They found  $a = 54·3$ , and  $b = 42 \times 10^3$ . The term  $b$ , which was negative at ordinary temperatures, became positive at liquid hydrogen temperature. The following results seem to lend a certain confirmation to Becquerel's theory.

*Hall Effect in Bismuth*

<i>H</i>	<i>RH</i> Obs.	<i>RH</i> Calc.	<i>H</i>	<i>RH</i> Obs.	<i>RH</i> Calc.
3450	$230 \times 10^3$	$229 \times 10^3$	9880	$583 \times 10^3$	$582 \times 10^3$
5660	350	352	11090	647·5	647
7160	431	434	12090	700	702
8520	503	507			

*Tellurium.* The Hall effect in *tellurium* is large, and the variation of the effect with temperature exhibits some interesting peculiarities. The effect was found by Ettingshausen and Nernst (274), 1886, to decrease with rise in temperature.

Onnes and Beckman <sup>(397)</sup>, 1913, measured the Hall effect in *tellurium* at ordinary and at very low temperatures. The results are given in the table below.

*Hall Effect in Tellurium*

<i>H</i>	Temperature	17° C.	-184° C.	-252.7° C.	-258.5° C.	$R_{20^0}/R_{290^0}$ Abs.
3750	Plate I	+39.1	—	+43.1	—	1.1
<i>H</i>	Temperature	18° C.	-184° C.	-252.7° C.	-258.5° C.	
3720	Plate II	+185.5	+210.5	+214.5	+211	1.1

The effect increases as the temperature falls. While the values of effects for the two plates vary widely, the ratio  $R_{20^0}/R_{290^0}$  Abs. is practically constant. The effect seemed

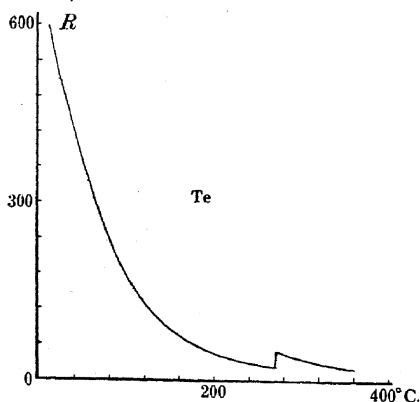


Fig. 52.

to be constant for all fields. However, for very low temperatures, there appeared to be a tendency for the effect to diminish with stronger fields.

The Hall effect in *tellurium* from 20.3° to 366° C. was measured by Smith <sup>(490)</sup>, 1913.

The curve in Fig. 52 gives the variation of the effect with temperature. The effect falls off rapidly with rise in temperature, until 180° C. is reached. Then the effect decreases very slowly until the temperature is slightly above 270° C., when the effect suddenly rises to twice its former value, then slowly decreases. This abrupt change in the effect was attributed by Smith to the fact that tellurium, as pointed out by Haken <sup>(233)</sup>, 1910, exists in two crystalline forms. One of the forms is stable below 350° C., while the other is stable above that temperature. In the

tellurium used by Smith the transformation temperature seems to be in the neighborhood of  $275^{\circ}\text{C}$ . This was attributed to the tellurium being a mixture of the two crystalline forms.

The variation of the Hall effect in *tellurium* with temperature was also determined by Wold <sup>(604)</sup>, 1916. He found, as indicated in the curve in Fig. 53, that the effect  $E$  (in microvolts) decreased rapidly with temperature, then changed sign, reached a negative maximum, decreased to zero again, became positive with a small value, which diminished at higher temperatures. Wold suggested that this peculiar behavior of the effect in tellurium is due to the presence of a mixture

of the  $\alpha$  and  $\beta$  crystalline forms of the metal, and that the  $\alpha$  form has a negative Hall effect, while the  $\beta$  form has a positive one, and that at certain temperatures the positive effect predominates, while at others the negative is

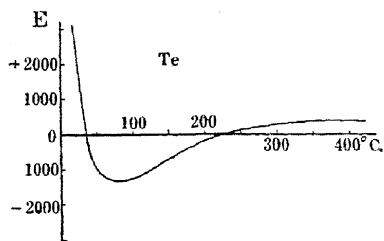


Fig. 53.

the greater. An analysis of the metal showed it to be very pure. Hence the peculiar variation of the effect cannot be attributed to the presence of impurities.

So long as the temperature remained constant, the effect was found to be independent of the magnetic field up to 12,000 gauss.

Wold endeavored to find the Hall effect in liquid tellurium, but without result. In finely powdered tellurium, in a form that approached the amorphous state, the effect was very small. These facts suggested to him that crystalline structure is essential to the presence of the Hall effect in metals. It is true that the effect in the highly crystalline metals, bismuth, antimony and tellurium is much greater than in the practically non-crystalline metals gold, platinum and copper.

*Antimony.* Drude and Nernst <sup>(154)</sup>, 1891, observed that the Hall effect in *antimony* decreased with rising temperature. Calling the effect at 17° C. unity, at 210° it was 0.78, and at 250°, 0.72.

Zahn <sup>(609)</sup>, 1904, found that *antimony* had a negative Hall temperature coefficient of  $-0.0017$ .

The following table, showing the variation of the Hall effect in *antimony* with temperature, is due to Barlow <sup>(44)</sup>, 1903.

*Hall Effect in Antimony*

Field	-186° C.	-79° C.	+215° C.	+58° C.
1750	+0.263	+0.249	+0.217	—
3960	0.252	0.243	0.211	—
6160	0.245	0.235	0.209	+0.203

The effect falls off gradually with rise in temperature, as well as with increase in field strength.

The Hall effect in *antimony* was determined at still lower temperatures by Alterthum <sup>(12-3)</sup>, 1912, with the following results:

-253° C.	-190° C.	-79° C.	+18° C.
+0.152	+0.163	+0.140	+0.125

From  $-253^{\circ}$  C. to  $-190^{\circ}$  C. there is an increase in the effect, then a steady decrease in the effect with rise in temperature. The latter variation of the effect is in accord with the determinations of Barlow above.

Bequerel, Matout and Wright <sup>(58)</sup>, 1913, found that the effect in *antimony* at liquid air temperature was greater than at ordinary temperature. The effect was measured in plates cut from crystals of antimony. In some cases the magnetic field was parallel and in others perpendicular to the principal crystal axis. The effect at liquid air temperature, when plotted against the field strength, exhibited some peculiarities. The curves in each case consisted of two straight line portions, connected by a curve of inflection. The curves cannot be

resolved into two simple components as in the case of bismuth, Fig. 42.

In general the Hall coefficient in *bismuth*, *tellurium* and *antimony* decreases with temperature from  $0^{\circ}\text{C.}$  to the melting point of the metal. At the latter temperature the effect would seem to be zero. At low temperatures *bismuth* may change sign and become positive, and with increase in temperature pass through a negative maximum.

*Other Non-Magnetic Metals.* The Hall coefficient in the non-magnetic metals not already treated varies very little with change in temperature.

Zahn <sup>(609)</sup>, 1914, working at temperatures between that of the room and  $100^{\circ}\text{C.}$ , detected no change in the Hall effect with temperature in *Co*, *Cu*, *Ir*, *Pd*, *Pt*, *Ag*, *Zn*, *C*, *brass*, *constantan*, and *German silver*.

In 1908, Frey <sup>(206)</sup>, employing the alternating current method, found that the Hall effect in *platinum* increased slowly from  $100^{\circ}$  to  $1500^{\circ}\text{C.}$ ; that the effect in *zinc* between  $80^{\circ}$  and  $400^{\circ}\text{C.}$  decreased slightly to a minimum, and slowly returned to its original value; that the effect in *gold* was practically constant from  $130^{\circ}$  to  $840^{\circ}\text{C.}$ ; and that the effect in *manganin* was nearly constant between  $150^{\circ}$  and  $800^{\circ}\text{C.}$

The table below gives the change of the Hall effect with temperature as determined by Smith <sup>(486)</sup>, 1910, in several non-magnetic metals.

*Change of Hall Effect with Temperature*

Metal	Temperature	$R$	Metal	Temperature	$R$
Platinum	$-190^{\circ}\text{C.}$	$-2.22 \times 10^{-4}$	Gold . . .	$-190^{\circ}\text{C.}$	$-7.25 \times 10^{-4}$
"	$+23^{\circ}$	$-2.03$	" . . .	$+25^{\circ}$	$-7.04$
Copper .	$-190^{\circ}$	$-6.5$	Zinc . . .	$-190^{\circ}$	$+10.9$
" .	$+23^{\circ}$	$-5.4$	" . . .	$+25^{\circ}$	$+6.3$
Palladium	$-190^{\circ}$	$-10.9$	Aluminium	$-190^{\circ}$	$-1.92$
"	$+23^{\circ}$	$-8.6$	"	$+25^{\circ}$	$-3.20$
Silver . .	$-190^{\circ}$	$-9.2$			
" . .	$+23^{\circ}$	$-8.4$			

The effect in all the above metals, except *aluminium*, diminishes with rise in temperature. The effect at  $-190^{\circ}\text{C}$ . was found to be independent of the field strength.

The changes of the Hall effect in *gold*, *silver*, *copper*, and *palladium* at very low temperatures were determined by Onnes and Beckman <sup>(395)</sup>, 1912, and are given in the following table.

*Hall Effect at Low Temperatures*

Temperature	Gold	Silver	Copper	Palladium
$17^{\circ}\text{C}$ .	$-7.24 \times 10^{-4}$	$-8.00 \times 10^{-4}$	$-4.92 \times 10^{-4}$	$-6.75 \times 10^{-4}$
$-252.7^{\circ}$	9.81	10.14	6.62	13.68
$-258.5^{\circ}$	9.82	9.91	6.56	13.85

In general the effect increases with fall in temperature. This change takes place chiefly between liquid air temperature,  $-190^{\circ}\text{C}$ ., and liquid hydrogen temperature,  $-252.7^{\circ}\text{C}$ . Below the latter temperature, the effect is practically constant.

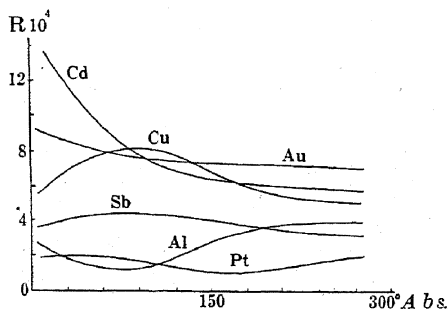


Fig. 54.

The curves in Fig. 54 exhibit the variation of the Hall effect with temperature, as determined by Alterthum <sup>(12-13)</sup>, 1912, in *gold*, *cadmium*, *aluminium*, *copper*, *antimony* and *platinum*. The effect in the diamagnetic metals *Au*, *Cd*, *Cu*, *Sb* reaches a maximum, while in the paramagnetic metals *Al*, *Pt*, the effect attains a minimum.

The following tables give the variation of the Hall effect in several substances with temperature, as determined by Onnes and Hof <sup>(398)</sup>, 1914.

*Hall Effect in Graphite (Compressed)*

Field	290° Abs.	20.5° Abs.	14.5° Abs.
	<i>R</i>	<i>R</i>	<i>R</i>
4800	+ 0.68	+ 3.4	+ 1.42
6000	+ 0.68	—	—
7200	—	+ 3.39	—
8400	+ 0.68	—	+ 1.52
9600	—	+ 2.81	—
11800	+ 0.74	+ 2.22	+ 1.52

The effect in this graphite is thus seen to increase as the temperature falls to about 20° Abs., and then decreases as the temperature is further lowered.

*Hall Effect in Cadmium (Rolled)*

Field	290° Abs.	20.5° Abs.	14.5° Abs.
3000	—	+ 13.1 × 10 <sup>-4</sup>	—
3600	+ 4.6 × 10 <sup>-4</sup>	—	—
4800	—	+ 20.2	+ 22.3 × 10 <sup>-4</sup>
6000	+ 6.3	—	—
7200	—	+ 20.6	+ 23.4
8400	+ 5.9	—	—
9600	—	+ 19.6	+ 22.3
11800	—	+ 17.6	+ 19.1
12000	+ 5.5	—	—

The Hall effect in *cadmium* is thus seen to increase greatly at low temperatures.

*Hall Effect at Helium Temperatures*

Metal	Field	4.25° Abs.	2.8° Abs.
Tin . . . . .	11300	+ 2.6 × 10 <sup>-5</sup>	+ 9.8 × 10 <sup>-5</sup>
Lead . . . . .	300	< 6 "	< 6 "
" . . . . .	1000	< 2 "	—
" . . . . .	5000	+ 0.8 × 10 <sup>-4</sup>	—
" . . . . .	11300	+ 1.8 "	+ 1.3 × 10 <sup>-4</sup>
Silver . . . . .	11300	— 16 "	— 16 "
Nickel . . . . .	1300	< 5 × 10 <sup>-5</sup>	—
" . . . . .	11300	< 1 "	< 1 × 10 <sup>-5</sup>
Bismuth . . . . .	1000	— 86.3	—
" . . . . .	5000	— 84.2	—
" . . . . .	11300	— 85.7	—



At hydrogen temperatures the Hall effect in *tin* and *lead* was too small to be measured. At helium temperatures, in fields so small that the two metals remained supraconducting, no Hall effect could be detected. Only when the field was great enough to set up ordinary resistance in these metals could the effect be measured. This latter fact emphasizes again the intimate connection between the Hall effect and the change in resistance in the magnetic field.

*Silver*, which is not supraconducting at helium temperatures, increased in its Hall effect as the temperature was lowered to  $4.25^{\circ}$  Abs., and then the effect remained constant as the temperature was further reduced.

The Hall effect in *nickel* at higher temperatures tends towards saturation. At hydrogen temperatures the effect is measurable, while at helium temperatures the effect has practically disappeared, although at these latter temperatures the metal is not supraconducting, and the resistance is still measurable.

The Hall effect in *silicon* was found by Buckley <sup>(104)</sup>, 1914, to decrease with temperature, between  $0^{\circ}$  and  $100^{\circ}$  C., in a field of about 9000 gauss. The change in the Hall effect did not correspond with the change in resistance.

#### 6. Reversal of the Hall Effect with Temperature

Hall <sup>(235)</sup>, 1880, and Ettingshausen and Nernst <sup>(174)</sup>, 1886, found the effect in *tin* to be negative at room temperature, while Onnes and Hof <sup>(398)</sup>, 1914, found that at  $4.25^{\circ}$  Abs. the effect was positive.

As already stated on p. 49, Lownds <sup>(345)</sup>, 1903, found the Hall effect in *bismuth* to be negative at  $16^{\circ}$  C. and  $-79^{\circ}$  C., but positive at  $-186^{\circ}$  C.

Also Onnes and Beckman <sup>(395)</sup>, 1912, p. 51, found that in a *bismuth* plate placed with its principal axis perpendicular to the magnetic field, the Hall effect was positive at the temperature of liquid hydrogen, while at  $17^{\circ}$  C. the effect was negative.



Van Everdingen observed that the change in resistance was least when the magnetic field was parallel to the principal axis. This latter fact probably accounts for the small Hall effect in the parallel position.

The variation of the Hall effect in a *crystal of bismuth*, when the primary current was parallel and perpendicular to the crystal axis in a field perpendicular to the axis, as determined by Lownds, has already been given on p. 50.

The fact, that the crystalline structure and the relative position of the crystal axis with respect to the primary current and the magnetic field profoundly influence the magnitude of the Hall effect, probably in many cases accounts for the discrepancies in the results of different investigators.

#### 8. Variation of the Hall Effect and Resistance

The Hall Effects and the specific resistances of substances have an extreme variation among themselves by as much as  $10^7$  times. However, the quotient  $\frac{R}{\rho}$  is fairly constant, as is seen in the table below, arranged by Königsberger and Gottstein<sup>(296)</sup>, 1913. The specific resistance is  $\rho$ , the Hall effect  $R$ .

Poor Conductors	$R$ at $18^\circ \text{ C.}$	$\rho$ at $18^\circ \text{ C.}$	$\frac{R}{\rho}$
$\text{MoS}_2$ . . . . .	$-2.6 \times 10^3$	1.8	$1.4 \times 10^3$
Si . . . . .	$+1.85 \times 10^2$	$4.4 \times 10^{-2}$	4.2 "
Graphite . . .	$-6.95 \times 10^{-1}$	$4.5 \times 10^{-4}$	1.5 "
$\text{Cu}_2\text{I}_2$ . . . . .	$+1.1 \times 10^{-4}$	94	$1.2 \times 10^3$
Metals			
Ag. . . . .	$-8.9 \times 10^{-4}$	$1.6 \times 10^{-6}$	$5.5 \times 10^2$
Au. . . . .	$-7.2$ "	2.3 "	3.6 "
Cu. . . . .	$-5.3$ "	1.7 "	3.1 "
Al. . . . .	$-3.9$ "	3.2 "	1.2 "
Pt. . . . .	$-2.0$ "	$1.1 \times 10^{-5}$	$1.8 \times 10^4$
Pd. . . . .	$-6.8$ "	1.1 "	6.2 "
Ir . . . . .	$+4.0$ "	$5.3 \times 10^{-6}$	8 "
Cd. . . . .	$+5.9$ "	7.6 "	7.8 "

Metals	$R$ at $18^{\circ}\text{C.}$	$\rho$ at $18^{\circ}\text{C.}$	$\frac{R}{\rho}$
Zn . . . . .	$+1.0 \times 10^{-8}$	$6.1 \times 10^{-6}$	$1.6 \times 10^2$
Fe . . . . .	$+1.0$ „	$1.0 \times 10^{-5}$	1 „
Ni . . . . .	$-2.8$ „	1.0 „	2.8 „
Sb . . . . .	$+1.2 \times 10^{-1}$	4.5 „	$2.7 \times 10^3$
Bi . . . . .	$-11$ ( $\perp$ H)	$1.2 \times 10^{-4}$	$1 \times 10^5$
„ . . . . .	$-0.4$ to $+10$ ( $\parallel$ H)	—	$4 \times 10^8$
Alloys			
Sb·Cd . . . .	$+8.01$	$5 \times 10^{-2}$	$1.6 \times 10^2$
Sb·Cd <sub>2</sub> . . . .	$+8 \times 10^{-1}$	$5 \times 10^{-3}$	1.6 „
German Silver	$-5 \times 10^{-4}$	$1.5 \times 10^{-5}$	$3 \times 10^1$
Constantan . .	$-9$ „	5 „	2 „

The values of  $\frac{R}{\rho}$  vary about  $10^2$  times. This points to the probable fact that the mean free paths of electrons in all metals are of about the same order of magnitude.

Königsberger and Gottstein <sup>(296)</sup>, 1913, found that, in the case of the poor conductors below, the ratio of the isothermal Hall effects at different temperatures,  $\frac{R_i}{R_i'}$ , was practically the same as the ratio of the electrons per unit volume at these temperatures.

*Hall Effect and Electron Number*

Substance	Temperature	$R_i$	$\frac{R_i}{R_i'}$	$\frac{N}{N'}$
Graphite . .	$16^{\circ}\text{C.}$	$R_i = -5.38 \times 10^{-1}$	} 1.11	1.14
„ . . . .	$90^{\circ}$	$R_i' = -4.85$ „		
Si (5.2% Fe)	$16^{\circ}$	+120	} 1.36	1.42
„ . . . .	$90^{\circ}$	+88		
MoS <sub>2</sub> . . . .	$18^{\circ}$	$-1.3 \times 10^3$	} 2.67	2.13
„ . . . .	$90^{\circ}$	$-4.9 \times 10^2$		

The Hall effect and the number of electrons diminishes with temperature at about the same rate. The values of  $N$  and  $N'$  are computed from the resistances.

## CHAPTER IV

## 1. Hall Effect in Compounds

The Hall effect in minerals, crystals, and chemical compounds has not been examined to any great extent, due in part to the difficulty of securing natural specimens of sufficient size and uniformity for successful manipulation.

The Hall effect in plates cast from molten *lead sulphide* was found by Van Aubel <sup>(28)</sup>, 1903, to have a negative sign and a small value compared with the effect in bismuth. The effect in *bismuth sulphide* <sup>(29)</sup> exhibited a strong dissymmetry and was of the same order of magnitude as the effect in bismuth.

The effect in *copper iodide* was observed by Baedeker <sup>(34)</sup>, 1909, to be positive, and to be independent of the magnetic field up to 6000 gauss.

The addition of iodine to the copper iodide decreased the resistance and also the Hall effect. In the following table is given the Hall effect for copper iodide whose resistance has been changed by the addition of iodine.

*Hall Effect in Copper Iodide*

$w \div w_g$	16	10.6	1.4
$R$	+6.23	+5.78	+0.24

In the above table  $w$  is the specific resistance of the copper iodide, while  $w_g$  is the specific resistance of the saturated condition.

Steinberg <sup>(505)</sup>, 1911, also determined the Hall effect in *copper iodide* at 18° C. and found the effect to be positive and independent of the field up to 7000 gauss. The effect was found to decrease with the iodine content, and to increase with the specific resistance, as shown in the table below.

*Hall Effect in Copper Iodide*

Specific Resistance	0.02	0.049	0.12	0.212	2.91	7.50	94.0
$R$	+ 0.550	+ 2.00	+ 6.90	+ 20.0	+ 340	+ 880	+ 11000

The Hall effect in one crystal of *galena* was found by Smith <sup>(490)</sup>, 1913, to have a value of  $-34.2$ , while the effect in another specimen had a value of  $-251$ . The Hall E. M. F. was found to be proportional to the magnetic field. The fact that Van Aubel (l. c.) found a very small Hall effect in *cast lead sulphide*, while in the crystal form a very large effect exists, again seems to point to an intimate connection between the Hall effect and crystalline structure. The Hall effect in *iron pyrites* was found by Smith (l. c.) to have a value of  $-0.296$ , and to be independent of the magnetic field.

Gottstein <sup>(218)</sup>, 1914, measured the Hall effect in *molybdenite* at  $18^{\circ}$  and at  $90^{\circ}$  C., and found it independent of the field strength. The effect decreased with rising temperature, as shown below.

*Hall Effect in Molybdenite*

Temperature	$R_i$	$R_a$	$\frac{R_a}{R_i}$
$18^{\circ}$ C.	- 1520	- 3030	1.99
$90^{\circ}$	- 481	- 878	1.83

The relation between the adiabatic effect,  $R_a$ , and the isothermal effect,  $R_i$ , above, is not a constant, but is represented by the Gans <sup>(208)</sup> equation,  $R_i = R_a \div \left(1.125 + \frac{q}{4T}\right)$ , where  $q$  is a constant for molybdenite and equal to 1020, and  $T$  is the absolute temperature.

**2. Hall Effect in Alloys**

The Hall effect in *copper-zinc* alloys was determined by Hall <sup>(211)</sup>, 1885. The results are given in the table below.

*Hall Effect in Copper-Zinc Alloys*

Plate	Cu %	Z %	Temperature	Field	R
A	100	0	21.5° C.	5560	-520 × 10 <sup>-8</sup>
B	81.3	18.7	25°	5750	-404 „
C	73	27	22.8°	5700	-250 „
D	67	33	25°	5930	-116 „
F	6	94	24° (?)	5690	+496 „
G	0	100	25°	6130	+830 „

If each metal in the alloy used had contributed its full share to the resultant Hall effect, the percentages would have been as follows:

Plates	A	B	C	D	F	G
Copper	100	91.3	80	73.6	24.3	0
Zinc . .	0	8.7	20	26.4	75.7	100

From a comparison of these percentages with those actually used, it is evident that the copper affects the magnitude of the Hall effect out of proportion to its content.

The Hall effects in the following table were determined by Ettingshausen and Nernst <sup>(173)</sup>, 1887.

*Hall Effect in Bismuth-Tin Alloys*

Field	Tin 0.0 %	Tin 0.95 %	Tin 1.46 %	Tin 6.14 %
2000	-9.84	-1.76	-0.69	-
6000	-7.20	-0.53	-0.07	+0.01
11000	-4.98	+0.12	+0.36	+0.09

From the above results it is seen that the presence of a small amount of tin reduces the Hall effect to an extraordinary extent, and even changes the sign of the effect, although at ordinary temperatures each metal has a negative coefficient.

The addition to *bismuth* of *antimony*, which has a positive Hall effect, was found by Van Aubel <sup>(26)</sup>, 1902; to greatly increase the Hall effect and leave it negative. In an alloy

of *bismuth* containing 8.3% of *antimony* the effect was found to be 3.4 times that of bismuth alone.

Zahn and Schmidt <sup>(616)</sup>, 1907, determined the Hall effect at 22° C. in *Heusler alloy* (Mn 25%, Al 15%, Cu 60%), and also in the constituents. The results are given in the table below. The values are approximate.

*Hall Effect in Heusler Alloy*

Metal	Heusler Alloy	Mn	Al	Cu
$R$	$+1300 \times 10^{-5}$	$-93 \times 10^{-5}$	$-40 \times 10^{-5}$	$-50 \times 10^{-5}$

Not only is the effect in the alloy much larger than in any constituent, but is of the opposite sign. The temperature coefficient was found to be 0.003. The effect decreased with increase in field. For 560 gauss,  $R = +0.0215$ ; while for 12,300 gauss,  $R = +0.0081$ .

Smith <sup>(487)</sup>, 1911, determined the Hall effect, electrical resistance and thermoelectric heights in *bismuth-antimony*, *antimony-cadmium*, and *antimony-zinc* alloys. In the first type of alloy the three effects increased with the addition of antimony, reached a maximum, when the antimony content was 10%, and then diminished rapidly on the further addition of antimony.

In the case of the other alloys examined by Smith, the maximum was a well-defined singular point, and corresponded to the compounds SbCd and SbZn.

Smith <sup>(490)</sup>, 1913, also determined the Hall effect in the magnetic alloys, *monel* metal (Ni 68%, Fe 1.5%, Mn 1%, Cu 29.5%), and *nichrome* (an alloy of Ni, Fe, Cr, Mn) and in *silicon steels*. In the alloys and steels the Hall effect was found to increase rapidly at first, and then to approach gradually a saturation value with increase of field strength. Variation of the effect was similar to that in the case of the magnetic metals. The Hall effect decreases in monel and nichrome as the temperature rises, while in the case of the silicon steels the effect increases with rise in temperature, as



it does in iron. The addition of small amounts of silicon greatly increased the Hall effect in steel. The following table gives the Hall effect in monel, nichrome, and silicon steel No. 1 (Si 3.8%).

*Hall Effect in Monel, Nichrome, and Silicon Steel*

Metal	Temperature	$H$	$R \times 10^3$	Metal	Temperature	$H$	$R \times 10^3$
Monel	25° C.	1650	-35.1	Steel I	19° C.	6000	+152
"	66°	"	24.2	"	130°	"	175
"	96°	3150	9.62	"	244°	"	208
"	137°	5300	2.38	"	310°	"	226
Nichrome	17°	3300	+199	K. Iron	17°	"	+10.8
"	120°	"	209	"	132°	"	22.4
"	198°	1050	200	"	250°	"	42.4
"	216°	"	113	"	321°	"	61.2
"	246°	3300	21.2				
"	286°	5550	8.6				

The Hall effect in K(ahlbaum) *iron* is given by way of comparison with the much larger value of silicon steel.

The Hall effect in *gold-silver* alloys was determined by Onnes and Beckman<sup>(395)</sup>, 1912, and by Beckman<sup>(54)</sup>, 1913, at low temperatures. The table below gives the results.

*Hall Effect in Gold-Silver Alloys*

Metal	Atomic % Ag	290° Abs.	90° Abs.	20.3° Abs.	14.5° Abs.
(Au	0	$-7.2 \times 10^{-4}$	$-7.6 \times 10^{-4}$	$-9.8 \times 10^{-4}$	$-9.8 \times 10^{-4}$
(Au-Ag) I	2.0	6.8	6.6	6.7	6.5
" II	10.7	5.6	5.25	3.7	3.7
" III	30.0	5.6	4.7	3.6	3.7
" IV	69.7	6.0	5.2	4.5	4.4
" V	90.9	7.35	6.45	5.75	5.75
" VI	97.8	7.7	7.4	6.95	6.9
Ag	100	8.0	8.2	10.15	9.9

Thus it is seen that on the addition of silver to gold, the Hall coefficient falls off until the alloy contains about equal quantities of the two metals, then the coefficient increases.

The Hall coefficient in *pure gold* and *pure silver* increased with fall in temperature, while in the *alloys* the coefficient decreased with fall in temperature.

The effect both in the pure metals and in the alloys was practically independent of the magnetic field.

The presence of *hydrogen* in *palladium* was found by Smith <sup>(490)</sup>, 1913, to reduce the Hall effect in that metal. In the regular metal alloys, the Hall effect is larger in those of high resistance. Since the presence of occluded hydrogen in palladium was found by McElfresh <sup>(349)</sup> to increase the resistance of the metal, one might suppose that the occluded gas would increase the Hall effect rather than diminish it. The presence of the gas may reduce the mean free path of the electrons, and thereby reduce the Hall effect.

The variation of the Hall effect in *tellurium-bismuth* alloys with the metal content was discovered by Trabacchi <sup>(545)</sup>, 1915, to have a striking resemblance to the variation of the thermoelectric height observed by Haken <sup>(233)</sup> in the same alloys.

The Hall effect in a series of magnetic alloys of *iron-copper*, *nickel-copper*, and *iron-nickel* was measured by Smith <sup>(496)</sup>, 1921, at room temperature, and in fields up to about 22,000 gauss. In all the alloys the Hall E. M. F. increased with the field, rapidly at first, and then in higher fields approached a state of saturation.

The following tables exhibit the change in the Hall effect with the change in composition of the alloys.

*Hall Effect in Magnetic Alloys*

Fe-Cu Alloys			Ni-Cu Alloys			Fe-Ni Alloys		
Fe %	Cu %	R	Ni %	Cu %	R	Fe %	Ni %	R
100	0.0	$+11.2 \times 10^{-8}$	100	0.0	$-7.65 \times 10^{-8}$	100	0.0	$+11.2 \times 10^{-8}$
99.196	0.804	12.2	95	5	29.8	98.93	1.07	16.9
98.49	1.51	21.0	92.5	7.5	40.9	98.07	1.93	17.8
98.0	2.0	19.0	90	10	52.0	92.95	7.05	42.5
96.0	4.0	16.3	85	15	59.1	91.83	8.17	43.1
93.84	6.16	15.6	75	25	83.7	89.80	10.20	52.0
92.95	7.05	15.2	70	30	41.9	86.89	13.11	61.5
			65	35	6.31	44.0	56.0	46.9
			50	50	1.33			

A small addition of copper, which has a negative Hall effect, to iron which has a positive effect, caused a rapid increase in the effect. The effect decreased on further addition of copper.

The addition of copper to nickel, both of which have a negative Hall effect, caused a rapid increase in the effect, which reached a maximum when the copper content was about 26%. The maximum effect was about 11 times that in pure nickel. Further addition of copper caused a decrease in the effect.

Small additions of nickel, which has a negative effect, to iron caused a rapid increase in the effect, which reached a maximum, which was about 6 times that in pure iron, and then decreased on further addition of nickel.

Intimate relations were found to exist between the Hall effect and the electrical resistance of the iron-copper alloys, and between the Hall effect and the thermoelectric height of the iron-nickel alloys.

*Hall Effect in Additional Alloys*

Alloy	Temperature	$H$	$R$	Observer
Brass . . . .	20° C.	Ind. H	Negative, Small	Zahn
Constantan .	"	"	— 0'0009	"
German Silver	Room temp.	10400	— 0'00053	Ettingshausen and Nernst
"	20° C.	5000 to 11000	— 0'00054	Zahn
Manganin . .	150°	4100	* 0'000569	Frey
" . .	440°	4790	0'000432	"
" . .	810°	4920	0'000460	"

\* Sign of the effect in manganin not given by the observer.

The above studies of the Hall effect in alloys show how greatly the presence of impurities may affect the magnitude of the effect in metals, and that, if the true effect is to be had, it must be sought in the pure metal.

### 3. Hall Effect in Dielectrics

In 1880, Hall<sup>(235)</sup> examined the action of the magnetic field on a dielectric in an electric field, but failed to observe

any change in the distribution of the equipotential lines in the dielectric.

In 1885, Van Aubel <sup>(20)</sup> also investigated the influence of the magnetic field on the electrostatic polarization in dielectrics. He likewise failed to detect any change brought about by the magnetic field.

According to Lorentz <sup>(338)</sup>, no such effect was to be expected, for in the case of an electrostatic charge the electrons are at rest, and hence create no magnetic field that can be affected by the electromagnet. Sir J. J. Thomson <sup>(517)</sup> showed that the ratio of the electromotive force, tending to displace the lines of electrostatic force, to the original electromotive force in the plate is given by the expression  $10^{-27} \dot{H} \sin \Theta$ , where  $\dot{H}$  is the rate of change of the magnetic force, and  $\Theta$  the angle between the electric displacement and the direction along which the change in  $H$  is greatest. Such a quantity as the above would be too small to observe, and furthermore, in the case of a constant magnetic field acting on an electrostatic charge, the expression vanishes.

The question naturally arises: Would the Hall effect be appreciable in a charged dielectric placed in an alternating magnetic field of very high frequency, or in a field rapidly changing in magnitude?

## CHAPTER V

### THEORIES OF THE HALL EFFECT IN SOLIDS

While no one of the theories thus far advanced explains fully and satisfactorily the Hall effect, it seems worth while to give in brief outline and in historical order the leading attempts to account for a phenomenon so fundamentally connected with the ultimate structure of matter.

## 1. Theory of Sir Wm. Thomson

Sir Wm. Thomson <sup>(530)</sup>, 1851, in his paper on the *Thermal Effects and the Thermo-electric Excitation of Electrical Currents in Homogeneous Crystalline Solids*, says: "A question now naturally presents itself: Are these three principal axes at right angles to one another in the substance, possessing the properties of symmetry with reference to thermoelectric qualities, analogous to those which have been established for the dynamical phenomena of a solid rotating about a fixed point, and for electrostatical and for magnetic forces, in natural crystals or in substances structurally crystalline as regards electric and magnetic inductions?"

"The following transformations suggested by Stokes's <sup>(510)</sup> [1851] paper on the *Conduction of Heat in Crystals* in which a perfectly analogous transformation is applied to the most general conceivable equations expressing a flux of heat in terms of variations of temperature along rectangular lines of reference in a solid, will show the nature of the answer.

"The body being crystalline, probably possesses different electrical conductivities in different directions, and the relation between current and electromotive force cannot, without hypothesis, be expressed with less than nine coefficients. These, which we shall call the coefficients of electric conductivity, we shall denote by  $\alpha, \lambda, \dots$ ; and we have the following equations, expressing by means of them the components of the intensity of electric current in terms of the efficient electromotive force at any point of the solid:

$$\begin{aligned} h &= \alpha \left( E - \frac{dV}{dx} \right) + \alpha' \left( F - \frac{dV}{dy} \right) + \alpha'' \left( G - \frac{dV}{dz} \right), \\ i &= \lambda'' \left( E - \frac{dV}{dx} \right) + \lambda \left( F - \frac{dV}{dy} \right) + \lambda' \left( G - \frac{dV}{dz} \right), \\ j &= \mu' \left( E - \frac{dV}{dx} \right) + \mu'' \left( F - \frac{dV}{dy} \right) + \mu \left( G - \frac{dV}{dz} \right).'' \end{aligned}$$

In a footnote added March 3, 1882, Thomson wrote:

"Hall's discovery in terms of the notation of formula (46)

{above equations], is simply that  $\lambda' - \mu''$ ,  $\mu' - \alpha''$ ,  $\alpha' - \lambda''$  are not each of them equal to zero for a metal in a magnetic field."

In another note, added at the same time, Thomson thus commented on Stokes's paper referred to above:

"In this paper Stokes explains the rotatory quality expressed by the general equations of thermal conduction, with nine distinct arbitrary constants as follows: 'Conceive an elastic solid to be fixed at the origin, and to expand alike in all directions and at all points with a velocity of expansion unity, so that a particle which at the end of time  $t$  is situated at a distance  $r$  from the origin, at the end of time  $t+dt$  is situated at a distance  $r(1+dt)$ . Conceive this solid to turn with an angular velocity of  $w$  equal to  $\sqrt{(w'^2 + w''^2 + w'''^2)}$  about an axis whose direction cosines are  $w'w^{-1}$ ,  $w''w^{-1}$ ,  $w'''w^{-1}$ . The direction of motion of any particle will represent the direction of the flow of heat in what we may still call the *auxiliary solid*, from whence the direction of the flow of heat in the given solid will be obtained by merely conceiving the whole figure differently magnified or diminished in three rectangular directions.

'This *rotatory* sort of motion of heat, produced by the mere diffusion from the source outwards, certainly seems very strange, and leads us to think, independently of the theory of molecular radiation, that the expressions for the flux with six arbitrary constants only, namely the expressions (8),

$$-f_x = A \frac{du}{dx} + F \frac{du}{dy} + E \frac{du}{dz},$$

$$-f_y = B \frac{du}{dy} + D \frac{du}{dz} + F \frac{du}{dx},$$

$$-f_z = C \frac{du}{dz} + E \frac{du}{dx} + D \frac{du}{dy},$$

are the most general possible.'

$[f_x, f_y, f_z$  are the fluxes across planes perpendicular to the axes  $x, y, z$ ;  $u$  is the temperature at a point  $P$ , and  $A, B, C, D, E, F$  six constants.]

"It appeared to me that the assumption made in this last clause was conceivably invalidated by the existence of crystals (such as those that have the 'Pyroelectric quality') with difference of crystalline form between two ends, the 'Dipolar' difference as I called it: and that it certainly was inapplicable to matter in a magnetic field, whether the matter be solid or fluid, or ferromagnetic or diamagnetic; that on the contrary we might expect to actually find the rotatory quality in thermal conduction under magnetic influence [Later, discovered by Leduc and by Righi]. The same considerations are of course applicable to electrical conduction, and accordingly I would not admit less than nine coefficients of electrical conduction. Hall's recent great discovery shows that the hypothesis which 28 years ago I refused to admit, was incorrect, and proves the rotatory quality to exist for electrical conduction through *métals* in the magnetic field."

At the meeting of the British Association at York, 1881, after Hall had presented an account of his recent discovery, Sir Wm. Thomson <sup>(238)</sup> said:

"The subject of this communication is by far the greatest discovery that has been made in respect to the electric properties of metals since the times of Faraday—a discovery comparable with the greatest made by Faraday. I look upon it with special interest myself as so closely connected with electrodynamic properties of metals, which formed the subject of my Bakerian Lecture in 1856. I pointed out in that paper, in about § 104, that it was to be expected that magnetic induction would produce change of thermal conductivity and of electrical conductivity in different directions in substances perfectly isotropic. I found by mathematical investigation rotational terms, and pointed out that we might expect in bodies, which have rotational quality, to find the effect of such terms exhibited. But the only influence having that relation to rotation which was necessary for producing the terms in question I pointed out to be the influence of a

magnet, and that we might expect that the effect of a magnet upon an isotropic body would be to induce difference of quality in different directions in accordance with the rotatory term, and I said I thought it improbable that the rotatory terms would be found to be null in a body subjected to the influence of a magnet. I look with great delight on Prof. Hall's discovery, as having verified that which I predicted as probable. I did not myself make any serious attempt to discover it. It is the first illustration ever brought out by experiment of one of the most curious and interesting things in the mathematics of æolotropy.

"The previous mathematical writers dismissed these terms altogether, although they found them in the formula;—dismissed them as something which we could not imagine to exist. I refused to dismiss them, and said there was decided reason that they could exist under the rotational influence which we know to belong to a magnet."

## 2. Maxwell's Equations and the Hall Effect

Hopkinson <sup>(267)</sup>, in 1880, called attention to the fact that the electromagnetic equations of Maxwell <sup>(362)</sup> contain a term that accounts for the Hall effect. For the flow of electricity through a body Maxwell obtained the following equations:

$$X = R_1 u + S_3 v + S_2 w - T v,$$

$$Y = S_3 u + R_2 v + S_1 w - T u,$$

$$Z = S_2 u + S_1 v + R_3 w,$$

in which  $X, Y, Z$  are the components of electromotive force,  $u, v, w$  the components of the current at any point, and  $R_1, R_2, R_3, S_1, S_2, S_3, T$  are constants depending on the substance conducting the electricity.

Maxwell said: "It appears from these equations that we may consider the electromotive force as the resultant of two forces, one of them depending on the coefficients  $R$  and  $S$ , and the other depending on  $T$  alone. The part depending



on  $R$  and  $S$  is related to the current in the same way that the perpendicular on the tangent plane of an ellipsoid is related to the radius vector. The other part, depending on  $T$ , is equal to the product of  $T$  into the resolved part of the current perpendicular to the axis of  $T$ ; and its direction is perpendicular to  $T$  and to the current, being always in the direction in which the resolved part of the current would lie, if turned  $90^\circ$  in the positive direction round  $T$ .

"Considering the current and  $T$  as vectors, the part of the electromotive force due to  $T$  is the vector part of the product  $Tx$  current. The coefficient  $T$  may be called the rotatory coefficient. We have reason to believe that it does not exist in any known substance. It should be found, if anywhere, in magnets which have a polarization in one direction, probably due to a rotational phenomenon in the substance."

Hopkinson remarked: "Does not the 'rotatory coefficient' of resistance completely express the important facts discovered by Mr. Hall? Instead of expressing these facts by saying that there is a direct action of a magnetic field on a steady current as distinguished from the body conducting the current, may we not with equal convenience express them by saying that the effect of a magnetic field on a conductor is to change its coefficients of resistance in such wise that the electromotive force is no longer a *self-conjugate*-linear-vector function of the current?"

The Hall effect then may be considered as due to a change in ohmic resistance brought about by the action of the magnetic field, such as would result if the molecules in the metal assumed a helicoidal arrangement about the magnetic lines of force. The usual form of Ohm's law,  $I = KE$ , would become  $I = KE + h(H \cdot E)$ , where  $I$  is the current,  $E$  the electromotive force,  $K$  the specific electrical conductivity,  $H$  the field strength, and  $h$  a constant on which the Hall coefficient depends.

### 3. Hall's Curving Ball Analogy

In 1880, Hall <sup>(235)</sup> in anticipation of the electronic theory of the flow of electricity in solids, wrote:

"Imagine now an electrical current to consist of particles analogous to a base ball, moving through a metal conductor, the electrical resistance of which will correspond to the mechanical resistance offered by the air. Suppose, further, the particles of electricity on coming within the influence of the magnet, to acquire a motion of rotation about an axis parallel to the axis of the magnet\*). Under all these supposed conditions we might perhaps expect to find the action which is actually detected. To account for the reversal of the action in iron, we might suppose particles to acquire in this metal a rotation about the same axis as in the other metals, but in the opposite direction. Even after all these generous concessions in favor of our hypothesis, however, it fails to account for the behavior of nickel as different from that of iron."

### 4. Mechanical Strain Theory of the Hall Effect

Bidwell <sup>(61-2)</sup>, in 1884, suggested that the Hall effect is due to thermo-electric currents set up by the magnetic field between strained and unstrained portions of the plate. Hall <sup>(241-3)</sup> has shown that this theory does not stand the test of actual experimental examination.

Tomlinson <sup>(541)</sup>, 1884, considered the Hall effect as intimately connected with the effect of mechanical stress on the specific resistance of the metal. The change in resistance is brought about by the strain set up in the effort of the metal to move across the magnetic lines of force. He found an analogy between a series of Hall coefficients and the variation of the specific resistance of the metals when stretched.

\*) Maxwell <sup>(362)</sup>: "I think we have good evidence for the opinion that some phenomenon of rotation is going on in the magnetic field; that this rotation is performed by a great number of very small portions of matter, each rotating on its own axis, this axis being parallel to the direction of the magnetic axis, etc."

Moreau <sup>(371)</sup>, 1900, also believed that the Hall effect was due to a mechanical strain set up in the metal plate by the magnetic field.

The mechanical strain theory, which is based on mass action, does not seem capable of explaining the intricacies of the Hall and the allied effects, which are concerned with inter- and intra-atomic action, and the movement of electrons.

### 5. Lorentz's Theory of the Hall Effect

Lorentz <sup>(338)</sup>, in 1883, developed the following equations for the flow of an electric current in a metal plate placed with its plane perpendicular to the lines of magnetic force. The density components of the electric current along the axes  $x, y, z$ , are  $u, v, w$ , the specific electric conductivity of the metal is  $K$ , and  $h$  an unknown factor, a function of the intensity of the field. When the magnetic field is on, the equations are,

$$u = -k \frac{\partial p}{\partial x} - hv, \quad v = -k \frac{\partial p}{\partial y} - hu, \quad w = -k \frac{\partial p}{\partial z},$$

$$\text{and } \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0,$$

where  $p$  is the potential at any point  $x, y, z$ , in the plate.

If the plate is thin, of thickness  $d$ , and is in the plane  $xy$ , and if the main current,  $I$ , flows along the axis  $x$ , then  $v=0$ , and  $w=0$ . The integration of the equations gives the electromotive force at the edges of the plate,

$$E = \frac{hI}{kd},$$

an equation in which the Hall electromotive force varies indirectly as the electrical conductivity of the metal.

Comparing the above equation with the usual Hall equation,  $E = \frac{RHI}{d}$ , we have  $h = RHK$ .

The above equations were developed from purely electrodynamic considerations. In 1905, Lorentz <sup>(340)</sup> proposed a new

theory to account for the motion of electrons under an electric and a magnetic force.

In this paper on the passage of electrons in metals, he remarked: "In the theory which admits of two kinds of free electrons, all questions relating to these phenomena become so complicated that I believe we had better in the first place examine more closely the Hall effect and allied phenomena. Perhaps it will be found advisable, after all, to confine ourselves to one kind of free electrons, a course in favor of which we may also deduce the results that have been found concerning the masses of the electrons. These tend to show that the positive charges are always fixed to ponderable atoms, the negative ones only being free in the spaces between the atoms or molecules. If, however, a study of the Hall effect should prove the necessity of operating with both positive and negative free electrons, we shall be obliged to face all the difficulties attending this assumption."

#### 6. Boltzmann's Theory of the Hall Effect

Starting with the Hall equations of Lorentz, Boltzmann<sup>(84)</sup>, 1880, deduced equations for special cases, which in reality predicted the effects discovered later by Ettingshausen and Nernst<sup>(171)</sup>, 1886.

Boltzmann also developed for the Hall electromotive force the equation,

$$E = w \frac{RH\gamma l I}{b(1 + \Phi)},$$

in which  $w$  is the resistance of the Hall circuit,  $\gamma$  the specific electrical conductivity of the metal;  $l$  and  $b$  the respective length and breadth of the plate;  $\Phi = w \div \frac{b}{\gamma l d}$ , in which  $d$  is the thickness of the plate; while  $R$ ,  $H$ ,  $I$ , have their usual significance.

Boltzmann also pointed out the fact, that to obtain the general theory of the Hall effect, the equations of Kirchhoff, Weber, Helmholtz, Maxwell, Stefan and others, for the motion

of electricity in conductors, must have a term added to represent the electrodynamic action that brings about the Hall effect.

### 7. Goldhammer's Theory of the Hall Effect

Lorentz's theory of the Hall effect does not take into account the change in resistance of the metal in the magnetic field. Goldhammer<sup>(214)</sup>, 1887, deduced equations for the electromotive force produced by the magnetic field, on the supposition that the metal was non-isotropic. In this case there are nine conductivity coefficients instead of one.

From the integration of the several equations, the following expression is found for the electromotive force set up by the magnetic field:

$$p = -\frac{p_1 - p_0}{l}x - \frac{\lambda_3}{\alpha_1} \cdot \frac{p_1 - p_0}{l}y - \frac{\lambda_2}{\alpha_1} \cdot \frac{p_1 - p_0}{l}z + \text{Const.}$$

The current is supposed to flow along the axis  $x$ , the plane of the plate is  $xy$ ,  $l$  is the length of the plate,  $p_1$  and  $p_0$  are the potentials of the electrodes,  $\alpha_1$ ,  $\lambda_2$ ,  $\lambda_3$ , are the coefficients of conductivity. The first term in the above equation represents the E.M.F. of the main current, the second is the Hall effect E.M.F., while the third is the E.M.F. along the axis  $z$ .

The Hall E.M.F. is then given by the equation

$$E = -\frac{\lambda_3}{\alpha_1} \cdot \frac{(p_1 - p_0)}{l} b,$$

where  $b$  is the width of the plate. The primary current is

$$I = \frac{(p_1 - p_0)}{\alpha_1 l} b d.$$

Hence

$$E = -\frac{\lambda_3}{d} I.$$

If we compare this equation with the usual Hall effect equation, p. 9, we shall have,  $-\lambda_3 = R.H.$

### 8. Riecke's Theory of the Hall Effect

Riecke<sup>(438)</sup>, 1898, assumed that an electric current consisted of positively and negatively charged particles travelling

in opposite directions in the conductor, and that these particles had both electrical and thermal movements.

For the Hall electromotive force set up in a magnetic field  $H$ , he deduced this expression:

$$E = \frac{v_p^2 w_n - v_n^2 w_p}{v_p w_n + v_n w_p} \cdot \frac{H b i}{\sigma},$$

in which  $v_p$  and  $v_n$  are the specific electric migration velocities,  $w_p$  and  $w_n$  the specific thermal migration velocities, respectively of the positive and negative particles,  $b$  the breadth of the plate,  $i$  the current density, and  $\sigma$  the specific conductivity of the metal. If  $w_p = w_n$ , the Hall coefficient becomes

$$R = \frac{1}{\sigma} (v_p - v_n).$$

For *bismuth* Riecke found—

$$v_p = 0.125 \times 10^{-5} \text{ cm. per sec. per 1 abs. E. M. F.}$$

$$v_n = 4.54 \times 10^{-5} \text{ „ „ „ „ „ „}$$

$$w_p = w_n = 0.215 \text{ cm. per sec.}$$

$$N_1 = 101 \times 10^{15} \text{ positive ions per c.c.}$$

$$N_2 = 170 \times 10^{15} \text{ negative „ „ „}$$

Since the probable number of atoms in 1 c.c. of *bismuth* is  $615 \times 10^{20}$ , only a small fraction of the atoms are mobile and take part in conduction, according to Riecke.

In 1906, Riecke<sup>(440)</sup> developed another expression for the Hall effect, viz:

$$R = \frac{1}{12} \cdot \frac{e}{c\alpha} \cdot \frac{v_1 l_1 - v_2 l_2}{\gamma T},$$

in which  $v_1$  and  $v_2$  are the velocities, and  $l_1$  and  $l_2$  the mean free paths of the positive and negative particles, and  $\gamma$  the electrical conductivity.

### 9. Drude's Theory of the Hall Effect

The theory for the Hall effect developed by Drude<sup>(149)</sup>, 1900, is based upon the supposition that electric and heat currents are due to the motion of both positively and negatively charged particles, and that the density of these

particles is a function of the temperature. For the Hall electromotive force he deduced the expression:

$$E \cdot \frac{d \log (N_1 N_2)}{dT} = \frac{H e e c}{\sigma} \left( v_1 \frac{d \log N_2}{dT} - v_2 \frac{d \log N_1}{dT} \right)$$

in which  $v_1$  and  $v_2$  are the migration velocities, and  $N_1$  and  $N_2$  the number respectively of the positive and negative ions per unit volume.

For *bismuth* Drude found—

$$N_1 = 0.69 \times 10^{19} \text{ positive ions per c.c.}$$

$$N_2 = 0.46 \times 10^{19} \text{ negative „ „ „}$$

These values are about 70 and 27 times those of Riecke, p. 79. Moreover, Riecke found  $N_1 < N_2$ , while according to Drude,  $N_1 > N_2$ .

According to the original theories of Riecke and of Drude, those metals, in which the negative particles predominate, have a negative Hall effect, while those metals, in which the positive ions predominate, have a positive Hall effect.

The results of the two theories, however, are contradictory for bismuth.

#### 10. De Heen's Theory of the Hall Effect

De Heen <sup>(253)</sup>, 1904, proposed to account for the different signs in the Hall effect by supposing that the magnetic field affected the rotation of the charged particles in the atom in different ways. The atom was supposed to consist of charged particles revolving about a common centre. The atoms of magnetic substances so orient themselves in a magnetic field that the charged particles, or electrons, revolve in the same direction as the Amperian current in the electromagnet; while the atoms of diamagnetic substances orient themselves in an opposite manner.

The influence of the magnetic field,  $H$ , on the rotation of the atom,  $A$ , and on the movement of the charged particles is given in Figs. 56, 57.

The inner circle represents the rotation of the atom,  $A$ , while the outer arrows give the revolution of the charged particles about the centre.

*Atom Rotation Positive*

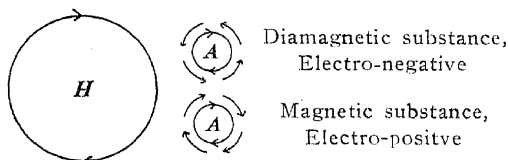


Fig. 56.

*Atom Rotation Negative*

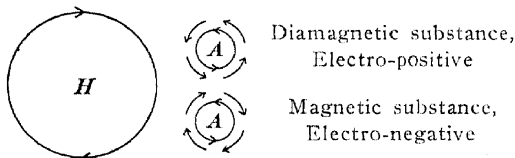


Fig. 57.

In accordance with this conception, the *iron* atom may be electro-positive, while the *nickel* atom is electro-negative, and in this way the difference in sign in the Hall effect in these two metals may be accounted for.

#### 11. Thomson's Free Electron Theory of the Hall Effect in Metals

In his corpuscular theory of the Hall effect, Sir J. J. Thomson<sup>(525)</sup>, 1907, supposed that between the atoms of a metal negative corpuscles, or electrons, are moving about freely, colliding with the atoms and rebounding, just as gas particles do. So long as the temperature remains the same, the mean kinetic energy of the electrons is constant. Since the electrons in a metal are moving in all directions, no electric current results from their motion. If, however, an electromotive force is impressed on the metal, the velocity of all the electrons will be changed slightly in the same direction, and since they are negatively charged, they will move in a



direction opposed to the electromotive force. The migration velocity of the electrons, to which the electric current is due, is small compared with the velocity of the electrons between collisions.

In a metal, in which the current is passing from left to right, Fig. 58, the negative particles are moving from right to left. When the plate is placed in a magnetic field  $H$ , whose lines of force are at right angles to the plate, the electrons moving with a velocity  $u$  from  $F$  to  $G$  will be acted upon by a force  $Heu$ , in which  $e$  is the electric charge on

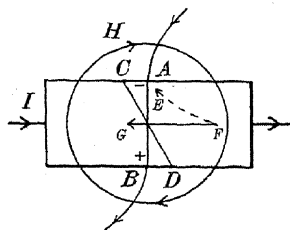


Fig. 58.

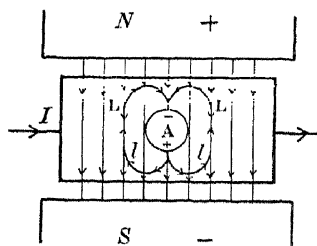


Fig. 59.

an electron. This force will tend to deflect the electron upward along the path  $FE$ . In consequence, the potential at the edge  $B$  will be raised and that at  $A$  lowered. In other words, the equipotential line  $AB$  will be rotated to  $CD$ .

If the above were a complete explanation of the action of a magnetic field on an electric current, the Hall effect would have the same sign (negative) in all metals. In the above consideration, no account was taken of the effect of collisions between electrons and atoms. It is conceivable that the magnetic field might cause the atoms to arrange themselves so as to produce a rotatory effect on the electrons when collisions occur. In the case of *iron*, the atoms are in themselves small magnets. In a magnetic field,  $NS$ , Fig. 59, the atoms will arrange themselves so that their negative poles are toward the positive pole of the magnet. Near the sides of an atom,  $A$ , the lines of force,  $l, l$ , of the atom magnet

are opposed to the lines of force,  $L, L$ , of the large magnet, while near the ends of the atom magnet the lines of force are in the same direction. Hence there will be some places near the atoms where the resultant field will be stronger than the external field. An electron on coming into collision with the atom might be rotated in a direction opposite to that given by the external field before collision.

This explanation would account for the positive sign of the Hall effect in *iron*, but would not account for the negative effect in *nickel*.

In the case of a diamagnetic substance, such as *bismuth*, the effect on the rotation of the electron due to collision would be the same as that brought about by the external field during the movement of the electron in its free path.

According to the above theory, the Hall effect consists of two terms. If the effect due to collision is greater than and opposite to that of the external magnetic field, the Hall effect will be positive. If the collision effect is less than, or in the same direction as that of the external field, the Hall effect is negative.

Thomson suggested that "it would be interesting to see if in exceedingly strong magnetic fields, much stronger than those required to saturate the *iron*, the Hall effect would change sign."

In fields as high as 24,000 gauss Smith<sup>(486)</sup>, 1910, observed that while the Hall effect in *iron* fell off, there was no indication of a reversal of the effect. Smith (l. c.), and Frey<sup>(206)</sup>, 1915, both noted that the effect in *iron* at temperatures beyond the critical fell off rapidly, but did not change sign.

## 12. Thomson's Electric Doublet Theory of the Hall Effect

Sir J. J. Thomson<sup>(525)</sup>, in his second theory of the Hall effect, dealt with the influence of the magnetic field on the distribution of the axes of doubly charged atoms moving under an electric force. The doublet atoms when not acted

upon by electric forces arrange themselves in the metal in a nondescript manner. When an electric force is impressed upon them, they align themselves parallel to the current, as shown in Fig. 60.

The negative charges  $e$  will travel from right to left, while the so-called positive current,  $I$ , will go from left to right.

Next consider the action of a doublet atom  $AB$  in the plane  $XOY$ , Fig. 61. The electric force acts along the axis  $X$ , and tends to turn the atom into a position parallel to  $OX$ . If the atom turns about the mid-point of its axis, the positive and negative charges will move with equal velocities, the

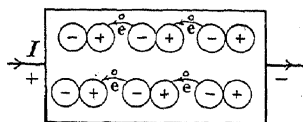


Fig. 60.

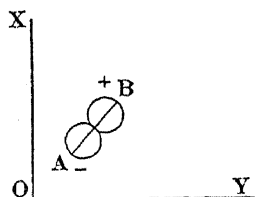


Fig. 61.

magnetic field will cause no deflection of the axis, and consequently no Hall current will be set up.

If, however, the atom rotates, for example, about the negative charge  $A$ , the magnetic force will cause the positive end  $B$  to dip below the plane  $XOY$ . In this way more negative ends may be above than below the plane, and a partial polarization will result at right angles to the plane  $XOY$ , similar to the arrangement of the atoms in Fig. 60. Hence a transverse electromotive force will be set up, a stream of negative electrons flowing upward, and the positive current downward and at right angles to  $XOY$ . This latter is the Hall current.

If the doublet atom under the electric force turns about the positive charge  $B$ , the magnetic field will cause the negative ends of the atoms to dip below  $XOY$ , and a Hall current of the opposite sign will result.

The sign of the Hall effect therefore depends upon whether the positive or negative charge on the atom moves more rapidly under the influence of the primary impressed electromotive force.

### 13. Gruner's Theory of the Hall Effect

Gruner <sup>(228)</sup>, 1908, assumed that in metals there existed only one kind of free electrons, the negative, together with positive and neutral molecules. The positive molecule may absorb a free electron when its velocity is below a certain value. If the velocity is above the critical, the impact between electron and molecule is a perfectly elastic one. In the steady state as many electrons are set free as are absorbed by the molecules.

With these assumptions, Gruner, following in the main the methods of Lorentz, deduced expressions for a number of galvanic and thermal phenomena.

The difference in sign of the Hall effect was attributed to the different ratios between the positive and neutral molecules in different metals.

### 14. Koenigsberger and Gottstein's Theory of Hall effect

Starting with the equation of Gans, p. 94,  $R_H = \frac{3\pi}{8eN}$ , Koenigsberger and Gottstein <sup>(296)</sup>, 1913, developed an expression to account for the change of sign in the Hall effect. They adopted the idea of Sir J. J. Thomson, p. 82, that the molecular magnets in solid substances assume some definite alignment under the influence of an external field, and that the magnetic field in the direction of the molecular axes is strengthened, while that perpendicular to the axes is weakened. This will be true so long as the distance between the molecules is sufficiently great, and their susceptibility is not far from 1. Such would be the case for rotating electrons of both para- and diamagnetic substances. In the case of strong molecular magnets, the internal molecular field might more

than compensate for the external field, and thus cause a change in the sign of the Hall effect.

The equation below, developed by the above authors, includes terms for the part played by both the external and the internal fields,  $H_a$  and  $H_m$ :

$$R \cdot H_a = \frac{3 \pi}{8 e} \left( \frac{N \cdot H_a + \int H_m dN}{N^2} \right), \text{ or}$$

$$R = \frac{3 \pi}{8 e N} (1 + \psi), \text{ where } \psi = \int \frac{H_m dN}{H_a N}.$$

This latter quantity may be positive or negative.

The authors stated that if the electrons in solids can pass from atom to atom along definite paths, or leave the atoms in a favored direction, then the Hall effect should indicate in which direction this passage takes place with respect to the molecular magnets.

#### 15. Richardson's Electron Theory of the Hall Effect

Richardson<sup>(433)</sup>, 1914, developed a theory for the Hall effect from a consideration of the motion of negative electrons under thermal agitation, under electric and magnetic forces, and between collisions. The electromotive force set up by the magnetic field  $H$  is given by the equation,

$$Y = \frac{2}{3} \cdot \frac{Hi}{ne},$$

in which  $i$  is the primary current,  $n$  the number of electrons in a unit volume,  $e$  the electric charge. In this equation all quantities are in electromagnetic units. The Hall coefficient is therefore

$$R = + \frac{2}{3} \cdot \frac{1}{ne}.$$

Since the sign of  $e$  is negative, this expression requires all Hall effects to be negative.

The value of  $e$  is approximately  $-10^{-20}$  E.M.U. The value of  $n$  has been estimated to be of the order of  $10^{20}$  to  $10^{24}$ . Accordingly  $R$  should be of the order of  $10^1$  to  $10^{-4}$ . The following are the Hall effects for a few metals.

Bi	Sb	Pb	Fe	Cu	Au
-10.27	+0.22	+0.00009	+0.0114	-0.0005	-0.00066

The above values are of the general order of  $10^1$  to  $10^{-4}$ .

Richardson <sup>(132)</sup> also suggested that the polarization of the electrons in metals might set up an influence in opposition to the external field that would bring about a change in the sign of the Hall effect.

### 16. Zahn's Theory of the Hall Effect

Zahn <sup>(613)</sup>, 1913, modified Drude's theory of the Hall effect by supposing that the vibratory and migratory velocity of the positive ions to be zero. On this basis he deduced this expression for the Hall coefficient:

$$R = \frac{1}{e N_1} \cdot \frac{\Phi_2}{\Phi_1 + \Phi_2}, \text{ in which}$$

$$\Phi_1 = \frac{d \log N_1}{dT}, \text{ and } \Phi_2 = \frac{d \log N_2}{dT} > < 0.$$

Here the effect varies inversely with the number of free electrons, and this number is a function of the temperature.

### 17. Livens' Electron Theory of the Hall Effect

Livens <sup>(320)</sup>, 1915, proposed a unitary electron theory to account for the regularities and irregularities of the Hall and allied effects. He followed in general the electron theories of Drude <sup>(150)</sup>, Thomson <sup>(520)</sup>, Lorentz <sup>(340)</sup>, Gans <sup>(208)</sup>, and Bohr <sup>(75)</sup>. His paper is mainly a rediscussion, modification and generalization of the above theories of metallic conduction and the Hall effect. According to his mathematical analysis, the Hall difference of potential is given by the equation,

$$E_z = \frac{-3\sqrt{\pi} \Gamma \left( \frac{3}{2} + \frac{4}{S} \right)}{4 N e c \left[ \Gamma \left( 2 + \frac{2}{S} \right) \right]^2} \cdot H I,$$

in which  $N$ ,  $e$ ,  $c$ ,  $H$ ,  $I$  have the usual significance, while  $S = qu^2$ ,  $q$  being a constant connected with the mean square of the

velocities of the electrons,  $u^2$ , or  $S$  is the constant of the force exerted by the molecules on the electrons.

The above expression agrees with that of Bohr and that of Gans, for the case they examined, where  $S = \infty$ .

According to the above equation the Hall effect should have the same sign in all metals. To account for the change of sign, Livens assumed, with Thomson and others, that the internal molecular magnetic field may be augmented by a local field due to a polarization of the electrons located on the molecules, and that this total inner field may so overcome the external field as to reverse the sign of the Hall effect.

In the *ferromagnetic metals* the local internal field varies directly as magnetization, and this internal field is probably the effective one, for it is known that the Hall effect in these metals varies as the magnetization and not as the external field. If the magnetization is substituted for the usual magnetic force  $H$ , the Hall coefficient for the ferromagnetic metals remains practically constant, as the field is increased.

In the simple *diamagnetic metals*, such as *gold*, *silver*, and *copper*, the internal local field is very small, and the Hall effect is practically constant for all fields. In these metals there is almost no variation of the Hall effect with temperature. This points to the fact that the action of a molecule on an electron is independent of temperature. The small change in the effect with temperature is probably due to a change in the number of electrons per unit volume.

In the *ferromagnetic metals* there is a considerable change in the magnitude of the Hall effect with temperature. The effect increases with permeability until the critical temperature is reached. Then it decreases rapidly and acquires a value of the order of that found in such metals as copper and gold.

In *bismuth*, and *other crystals*, the Hall effect varies with the direction of the magnetic field with respect to the crystalline axis. This is probably due to the fact that in a crystal the molecules have already a definite linear and axial

arrangement, and in consequence the internal field, due to the motion of the electrons in and about the molecules, varies in different directions.

Later, Livens<sup>(330)</sup>, 1918, deduced the following expression for the Hall electromotive force:

$$E = \frac{-3\pi}{8Nec} \cdot BJ,$$

in which  $N$ ,  $e$ ,  $c$  have the usual significance,  $B$  is the magnetic induction, and  $J$  the primary electric current.

In non-magnetic substances,  $B = H$ .

### 18. Hall's "A Possible Explanation of the Hall Effect"

In 1915, Hall<sup>(245)</sup> proposed the following as a possible explanation of the Hall effect. In 1914, in his paper on electric conduction and thermoelectric action<sup>(244)</sup>, Hall had assumed that conduction is maintained in part by free elec-

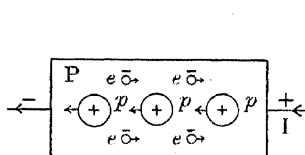


Fig. 62.

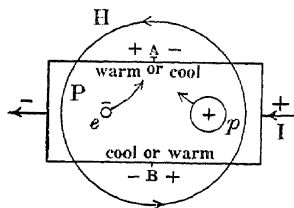


Fig. 63.

trons, which behave like gas molecules in the spaces between the atoms of the metal, and in part by electrons that go from atom to atom during collisions, without affecting the gas pressure action of the free electrons. The part played by the free electrons in conduction he considered unimportant, while in thermoelectric phenomena their part is all-important. The metal, or conducting body, was supposed to be made up of normal or neutral atoms, of positive atoms that had lost an electron, and of free electrons. The number of positive atoms or ions and of negative electrons was supposed to be the same.

When a current  $I$  of electricity passes through a plate  $P$ , Fig. 62, the electrons,  $e$ ,  $e$ ,  $e$ ,  $e$ , will move to the right, while



the positive ions,  $p, p, p$ , will move through a limited space to the left. If the plate is placed in a magnetic field  $H$ , Fig. 63, the electron  $e$  and the positive ion  $p$  will both be urged toward the upper edge of the plate. The electron is mechanically free to move up to the edge of the plate, while the atom can only change the direction of its vibration. The upward movement of the electrons tends to produce a negative charge at the edge of the plate, while the new movement of the positive ions constitutes a positive current that tends to produce a positive charge at the same upper edge.

To account for the fact that the Hall effect is positive in some metals and negative in others, it would only be necessary to say that in metals of the former class the tendency of the positive ions to move under the magnetic field is stronger than that of the electrons, and the upper edge becomes positive; while if the action of the electrons prevail, the upper edge will become negative.

In the former case, in which there is a positive charge at the upper edge of the plate, there will be a continual passage upward of free electrons, and there must also be a downward movement of equal number of electrons that are naturally not in a free state. This latter movement is brought about by the action of the vibrating atoms of the metal, which move upward a short distance as positive ions unite with electrons, and return downward a bit as neutral atoms. That is, there occurs a constant dissociation of the neutral atoms into electrons and ions at the lower part, and a recombination of the electrons and ions at the upper part of the plate. The force that carries the electrons and neutral atoms downward is termed an elastic-mechanical one.

The dissociation and recombination are brought about in the following manner. When the free electrons strike against the atoms of a metal, a distribution of thermal velocity takes place among these electrons. The slower, or *cold*, electrons would move upward under the action of the magnetic field,

and would cool the upper edge of the plate. Since dissociation of the neutral atoms increases with temperature, the cooling of the upper edge would bring about a recombination of electrons and positive ions. This recombination would produce heat. If this latter action is stronger than the cooling effect of the cold electrons that flow upward, then the upper edge of the plate will be warmed. If the influence of the cold electrons prevail, the upper edge will be cooled. In this way the difference in signs of the Ettingshausen effect (p. 147) may be accounted for.

### 19. Borelius' Diamagnetic and Paramagnetic Hall Effect

Borelius<sup>(93)</sup>, 1919, in developing his expressions to account for the different signs of the Hall effect employed the space-lattice conception of the structure of metals. He supposed the metal to consist of an equal number of positive atoms, and of negative electrons equally spaced between the atoms. The motion of the atoms to and fro and the magnetic field cause the electrons to have both a radial and a circular motion, giving rise to two susceptibilities, radial and circular. The relative value of these two determines whether a metal is paramagnetic or diamagnetic.

These two motions of the electrons give rise to two components of the Hall effect. For the diamagnetic Hall effect, Borelius deduced this expression:

$$R_d = + \frac{3}{4} \frac{e}{en} q,$$

in which  $e$  is the electronic charge,  $q$  is a function of the susceptibility, and  $n = \frac{N\rho}{A}$ , where  $N$  is the Avogadro number,  $\rho$  the density, and  $A$  the atomic weight of the metal. This diamagnetic component of the Hall effect is linear and independent of the field.

The paramagnetic component of the effect is given by the equation:

$$R_p = \frac{F(\psi)}{en}, \text{ where}$$

$$F(\psi) = -\frac{3\pi}{8} \left( \psi \sin \psi + \ln \psi - \frac{\sin \psi}{\psi} \right),$$

in which  $\psi$  is an angle connected with the path of the free electron, and depends upon the field.

The total Hall effect then is

$$R = R_d + R_p = \frac{1}{en} \left[ \frac{3}{4} q + F(\psi) \right].$$

This latter expression may change sign in accordance with the relative values of the two susceptibilities of the metal and the value of  $\psi$ , and in this way the two signs for the Hall effect are accounted for.

Borelius pointed out that for metals other than bismuth, antimony, and the ferromagnetic ones, the expression  $\frac{Rq}{A}$  is fairly constant, as shown in the table below.

Metal	$R > 10^4$	$\frac{Rq}{A} > 10^4$	Metal	$R > 10^4$	$\frac{Rq}{A} > 10^4$
Na . . .	-25	-1.07	Ag . . .	-8.2	-0.80
Mg . .	-9.4	-0.67	Au . . .	-7.2	-0.71
Al . . .	-3.6	-0.35	Zn . . .	+6.3	+0.68
Ir . . .	+3.8	+0.45	Cd . . .	+5.7	+0.44
Pd . . .	-6.8	-0.76	Sn . . .	-0.4	-0.02
Pt . . .	-1.7	-0.18	Pb . . .	-0.9	+0.05
Cu . . .	-5.2	-0.73			

The poor agreement in the case of tin and lead may be due to the very small Hall effect in these metals, and hence the presence in their determination of large probable errors.

## 20. Additional Theories of the Hall Effect in Solids\*)

Among the additional theories of the Hall effect in solids, the following may be mentioned, as worthy of consideration:

Those of Venske<sup>(553)</sup>, 1888; Lommel<sup>(335)</sup>, 1892; Wind<sup>(599)</sup>, 1896; Voigt<sup>(560)</sup>, 1898; Van Everdingen<sup>(187)</sup>, 1901; Kolářček<sup>(294)</sup>,

\*) See also Addendum p. 270.

1904; De Thullie <sup>(537)</sup>, 1912; Birkeland <sup>(69)</sup>, 1912<sup>\*</sup>; Righi <sup>(452)</sup>, 1915. The above theories are more or less mathematical.

According to most of the theories thus far advanced, the Hall effect should have the same sign in all metals. Fournier <sup>(204)</sup>, 1916, endeavored to account for the positive Hall effect in such metals as antimony, iron and tellurium, in the following way. He claimed that metallic conduction is not carried on by the free electrons alone. In metals, such as the above, there is probably only a very small amount of dissociation into free electrons and positive atoms. The negative electrons, as ions in electrolytic dissociation, gather neutral atoms about them. These weighted electrons are probably less mobile than the positive atoms. While heat causes the splitting off of electrons from bismuth atoms, this does not seem to be the case with iron. The atoms of iron, however, possess considerable innate mobility, since they can arrange their axes parallel to magnetic lines of force. Iron is also trivalent and can hold several electrons bound. In iron, then, it may be that most of the electrons are held bound, while some of the positive atoms are free to move, and by their superior number make up for lack of mobility.

Metals, then, in which most of the electrons are held bound and the positive atoms possess a large total mobility, should have a positive Hall effect, while metals in which the free negative electrons predominate should have a negative effect.

None of the monistic or dualistic theories, thus far advanced, account fully and satisfactorily for the two signs in the Hall effect. Subsequent theories of the Hall effect should include a consideration of the influence of the external magnetic field on the orientation of the orbits of revolution of the electrons, and on the manner in which the electrons are sent forth from the parent atom. The influence of the inner atomic magnetic field on the motion of electrons, and on that of positive and neutral atoms, must be taken into account.

A clearer understanding of the space-lattice structure of the metals should aid in the formulation of an adequate theory to account for the anomalies in the Hall effect.

Hall <sup>(237)</sup>, in 1881, in one of his earlier papers, remarked: This difference of sign in the rotational coefficients of the magnetic metals is so anomalous and so important a fact, that one returns again and again to its consideration."

Now after more than four decades, the above words are still pertinent. A satisfactory explanation of the variation in signs of the Hall and allied effects waits upon the future.

### 21. Isothermal and Adiabatic Hall Effects

Ettingshausen <sup>(169)</sup>, 1887, observed that the Hall effect is accompanied by a change of temperature at the edges of the metal plate where the secondary electrodes are fastened. If this difference of temperature is eliminated by a flowing bath, by the alternating current method, or otherwise, the Hall effect obtained under such conditions is termed *isothermal*. The Hall effect measured in the presence of the Ettingshausen temperature difference is called *adiabatic*.

Gans <sup>(208)</sup>, 1906, developed equations for the isothermal and adiabatic Hall effects, in which only the movements of negative electrons were considered. For the isothermal transverse difference of potential he obtained

$$E_i = \frac{3\pi}{8} \cdot \frac{ev}{\sigma} \cdot H j,$$

in which  $e$  is the charge on an electron,  $v$  the migration velocity of the electrons,  $\sigma$  the electrical conductivity of the metal,  $H$  the field strength, and  $j$  the primary current. Since  $Ne^2v = \sigma$ , we have for the isothermal Hall coefficient

$$R_i = \frac{3\pi}{8eN},$$

in which  $N$  is the number of electrons per unit volume. For the adiabatic effect, the following equation was obtained,

$$E_a = \frac{3\pi}{16} \cdot \frac{b+q}{4} \cdot \frac{ev}{\sigma} \cdot Hj, \text{ in which } b = 2T \frac{d \log N}{dT},$$

and  $T$  is the absolute temperature. From the above we have

$$R_a = \frac{3\pi}{16} \cdot \frac{b+q}{8} \cdot \frac{1}{eN}.$$

$$\text{Hence } R_a \div R_i = \frac{b+q}{8}, \text{ or } R_i = R_a \div \left(1.125 + \frac{q}{4T}\right),$$

where  $q$  is the heat of dissociation.

Gans (l. c.) determined the Hall effect in *bismuth* by measuring the difference of potential with a quadrant electrometer. This effect he termed the *electrometric Hall effect*.

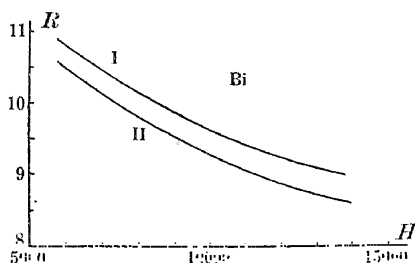


Fig. 64.

He also measured the Hall effect by the null-galvanometer method. This effect he called the *galvanometric Hall effect*.

Curve I in Fig. 64 represents the change of the galvanometric effect with field strength, while curve II gives the variation of the electrometric effect. The latter effect is consistently smaller.

Corbino <sup>(127)</sup>, 1911, called attention to the fact that Riecke and Drude in their theories did not take into account the influence of the Ettingshausen effect on the Hall effect. Allowing for this influence, he developed the expression below for the *isothermal* Hall effect, based on the usual assumptions for the two kinds of electrons:

$$R_i = \frac{c}{e} \cdot \frac{N_1 v_1^2 - N_2 v_2^2}{(N_1 v_1 + N_2 v_2)^2},$$

in which the letters have their usual significance.

For the Hall coefficient, corrected for the Ettingshausen effect, Corbino obtained this expression,

$$R_{corr.} = \frac{c}{e} \cdot \frac{1}{N_1 - N_2}.$$

In order to eliminate the Ettingshausen effect, Senepa<sup>(479)</sup>, 1913, placed the plate of *bismuth* in a bath of freely flowing water. He measured the isothermal effect at 16° C., 24° C., and 32° C., and found that the effect decreased with rise in temperature, but was independent of the field.

To determine what he termed the *corrected Hall effect*, Senepa sent through a *bismuth* plate a heat current in the proper direction with respect to the electric current, and of such a value, that in the magnetic field the Ettingshausen difference of temperature was annulled by the Righi-Leduc effect (p. 233).

The corrected Hall effect thus found was  $R_{corr.} = -7.78$ , at 43° C., while that found by the water bath method was  $-7.35$  at 44° C. From these results, and by means of Corbino's equations, he computed these values:

$$\begin{aligned} N_1 &= 0.24 \times 10^{18}, & N_2 &= 8.02 \times 10^{18}; \\ v_1 &= 1.57 \times 10^{15}, & v_2 &= 3.75 \times 10^{15}. \end{aligned}$$

Hence in bismuth the negative particles, or electrons, are 33 times as numerous as the positive particles, and the mobility of the negative is more than twice that of the positive.

In variable conductors, such as *silicon*, *graphite*, and *molybdenite*, in which the number of free electrons changes greatly with temperature and under other influences, according to Gottstein<sup>(218)</sup>, 1914, the isothermal Hall effect must be corrected for the Peltier and Nernst effects as follows:

$$R_i (corr.) = R_i + \frac{2 \Pi Q}{k},$$

in which  $\Pi$  and  $Q$  are the Peltier and Nernst coefficients respectively, and  $k$  is the thermal conductivity.

The following table gives the isothermal effects as determined by Koenigsberger and Gottstein <sup>(297)</sup>, 1915, at ordinary temperature.

*Isothermal Hall Effects*

Metal	$R_i$	Metal	$R_i$
Ag . . . . .	$-8.9 \times 10^{-4}$	Fe . . . . .	$+1.0 \times 10^{-3}$
Cu . . . . .	$-5.3$ „	Ni . . . . .	$-1.0$ „
Graphite . . . . .	$-4.9 \times 10^{-1}$	Sb . . . . .	$+2 \times 10^{-1}$
Zn . . . . .	$+1.0 \times 10^{-3}$	Si . . . . .	$+1.2 \times 10^{-2}$
Pd . . . . .	$-6.9 \times 10^{-4}$	MoS <sub>2</sub> . . . . .	$-1.3 \times 10^{-3}$

The adiabatic and isothermal Hall effects were determined for *antimony*, *bismuth*, and *bismuth-tin alloys* by Zahn <sup>(615)</sup>, 1915. The alternating current method was used for measuring the isothermal effect. The effect was measured both while the plate was in running water and in air.

The isothermal effect in *antimony* was found to be  $R_i = +0.22$ , which was very little less than the adiabatic effect.

The results for bismuth are given in the table below.

*Isothermal and Adiabatic Hall Effects in Bismuth*

Field	2500	4510	7500	9070	10100
$\Delta T$ in air . . .	0.082° C.	0.196°	0.340°	0.424°	—
$\Delta T$ in water . .	—	0.073°	0.133°	0.163	—
$R_i$ . . . . .	— 1.89	— 1.81	— 1.47	— 1.32	— 1.24
$R_a \div R_i$ in air .	1.046	1.050	1.052	1.054	—
$R_a \div R_i$ in water	1.035	1.038	1.048	—	—
$P \times 10^5$ . . . . .	0.80	1.09	1.17	1.18	—
$P \times 10^5 \div R_i$ . .	0.42	0.61	0.80	0.91	—

The Ettingshausen effect is represented by  $P$ .

The temperature difference at the secondary electrodes is much reduced, but not entirely eliminated by the water.

## 22. Bearing of the Hall Effect on the Theory of Magnetism

Kolářek <sup>(293)</sup>, 1895, Whittaker <sup>(578)</sup>, 1910, and others have considered the bearing of the Hall effect on the question whether magnetism is a linear or a rotatory phenomenon.



If magnetism is supposed to be linear, then an electric current must be rotatory. Suppose a current,  $I$ , flows from left to right in a plate, Fig. 65, placed at right angles to the field,

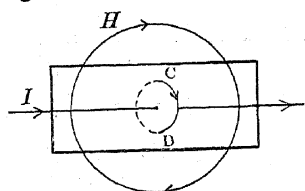


Fig. 65.

$H$ , and that the current rotates in a plane,  $CD$ , perpendicular to the plate. Under these conditions there is nothing to determine the direction of the secondary or Hall current. Thus it would seem that the existence of a Hall effect

is inconsistent with the supposition that magnetism is a linear phenomenon, and that magnetism must be rotatory in nature.

### 23. Relation Between the Hall Effect and Thermoelectric Power

It was first pointed out by Beattie <sup>(50)</sup>, 1896, that the Hall effect and the thermoelectric power of metals have corresponding signs. He arranged the table below, in which the metals are placed in the thermoelectric series according to Wiedemann (*Lehre von der Elektrizität*, Vol. II.) and the Hall coefficients, determined by Ettingshausen and Nernst <sup>(174)</sup>, are placed opposite.

*Thermoelectric Series and Hall Effect*

Metal	$R$	Metal	$R$
Bi . . . . .	- 10.1	Pt . . . . .	- 0.00024
Co . . . . .	+ 0.0046	Au . . . . .	- 0.00071
Ni . . . . .	- 0.0242	Ag . . . . .	- 0.00083
German Silver	- 0.00053	Zn . . . . .	+ 0.00041
Pd . . . . .	- 0.00115	Cd . . . . .	+ 0.00055
Al . . . . .	- 0.00038	Fe . . . . .	+ 0.0113
Pb . . . . .	+ 0.00009	Sb . . . . .	+ 0.192
Sn . . . . .	- 0.00004	Te . . . . .	+ 532.0
Cu . . . . .	- 0.00052		

The only two irregularities as to direction are cobalt and lead.

The magnitude of the Hall effect does not follow the order of the thermoelectric series where the effect is very

small. This may be due to the fact that the thermoelectric power and the Hall effect were determined in different specimens, whose difference in structure and composition may account for the seeming exceptions. The metals that occupy extreme positions at either end of the thermoelectric series also have extreme positions with respect to the Hall effect.

Beattie found in alloys of *bismuth-lead*, *bismuth-lead-antimony*, and *antimony-cadmium*, that the alloys with extreme thermoelectric position in the series had also a large Hall effect. He made a comparison of the variation with temperature of the Hall effect in *bismuth*, as determined by Lebreton<sup>(314)</sup>, 1895, with the variation of the thermoelectric force of *bismuth-lead* couples according to Dewar and Fleming<sup>(143)</sup>, 1893, and showed that the curves are strikingly similar. Koenigsberger and Gottstein<sup>(295)</sup>, 1913, confirmed the truth of the principle laid down by Beattie as to the relation between the thermoelectric power and Hall effect in a large series of metals and compounds. The reversal of the Hall effect and the change of thermoelectric property of metals in a magnetic field they attributed to electrons entering the magnetic field of an atom, and rebounding in a semi-circular path, thus changing a positive electronic pressure into a negative one.

The following table contains the thermo-E. M. F. of metals with respect to lead and the Hall effects in metals arranged in their order of magnitude. In general the Hall effect of an element increases with its thermoelectric height.

*Hall Effect vs. Thermo-E. M. F.*

Metal	<i>R</i>	E. M. F. in E. M. U.	Metal	<i>R</i>	E. M. F. in E. M. U.
Sn . . .	— 0.00004	— 17	Al . . .	— 0.00038	— 50
In . . .	— 0.000073	+ 240	Ir . . .	+ 0.000402	— 244
Pb . .	+ 0.00009	0	Cu . .	— 0.00052	+ 282
Pt . . .	— 0.000127	— 304	Cd . .	+ 0.00055	+ 302
Tl . . .	+ 0.00024	+ 214	Au . .	— 0.00066	+ 278

Metal	$R$	E.M.F. in E.M.U.	Metal	$R$	E.M.F. in E.M.U.
Ag . .	-0.00083	+230	Ce . .	+0.0019	—
Mg . .	-0.00094	-12	Na . .	-0.0025	-416
Ta . .	+0.00101	-130	Ni . .	-0.0047	-1907
Zn . .	+0.00104	+270	Fe . .	+0.0108	+1584
Pd . .	-0.00115	-700	C . .	-0.17	+1165
W . .	+0.00118	-120	Sb . .	+0.186	+149
Mo . .	+0.00126	—	Bi . .	-4.82	-7450
Co . .	+0.0016	-2042	Si . .	+41.00	-40,000
Li . .	-0.0017	+1150	Te . .	+536.0	+50,000

The E.M.F. is considered positive when the current goes from the metal to lead through the cold junction.

## CHAPTER VI

### 1. Hall Effect in Liquid Metals

Drude and Nernst <sup>(154)</sup>, 1890, placed a thin layer of molten *bismuth* between glass into which the electrodes were sealed, and found that a magnetic field produced a difference of potential at the secondary electrodes. This effect was reversed with the reversal of the field, but not with the reversal of the primary current. Hence it was not a true Hall effect. The same phenomenon was observed in mercury.

While examining the change of resistance in *mercury* in a magnetic field, Des Coudres <sup>(132)</sup>, 1891, incidentally discovered an effect in *bismuth amalgam* and in *mercury*, which he attributed to the presence of a Hall effect. His results are not, however, free from doubt.

Amaduzzi and Leone <sup>(16)</sup>, 1900, observed in *bismuth amalgams* phenomena which they considered Hall effects. They found no effect in liquid *mercury*.

Amerio <sup>(17)</sup>, 1901, examined *mercury* and liquid *amalgam* of *bismuth* for the Hall effect. He found a difference of potential, which was reversed on the reversal of the field.

The reversal of the primary current brought about a change of value, but no reversal in sign of the difference of potential.

Amerio considered that the phenomenon observed by himself and others could not be a true Hall effect, could not be due to a thermomagnetic effect or to a change of resistance in the magnetic field, nor to a thermoelectric effect where the electrodes joined the amalgam or mercury. He attributed the phenomenon to a displacement in the conducting layer between the principal electrodes, a displacement that follows Ampère's rule for a conductor in a magnetic field.

Von Traubenberg<sup>(548)</sup>, 1905, as already noted (p. 50), found the Hall effect in *bismuth* disappeared at the melting point of the metal, while in other metals the effect has been found to diminish greatly at very high temperatures, suggesting a disappearance of the effect at their melting points.

The Hall effect in liquid and in solid *mercury* has been recently investigated by Fenninger<sup>(197)</sup>, 1914. He concluded that the effect in liquid mercury could not be greater than 0.00002, in fields from 10,000 to 22,400 gauss; and in fields from 10,000 to 12,100 gauss the effect in solid mercury could not exceed 0.000011.

Up to the present (1923) the existence of a Hall effect in liquid metals has not been satisfactorily established.

## 2. Hall Effect in Electrolytes

The Hall effect in electrolytes has been investigated by Roiti<sup>(456)</sup>, 1882; Bagard<sup>(37)</sup>, 1896; Florio<sup>(201)</sup>, 1896; Chiavassa<sup>(116)</sup>, 1897; Everdingen<sup>(181)</sup>, 1898; Amaduzzi and Leone<sup>(16)</sup>, 1900; Moretto<sup>(379)</sup>, 1902; Amerio<sup>(17)</sup>, 1901; Heilbrun<sup>(254)</sup>, 1904; Oxley<sup>(401)</sup>, 1913; Righi<sup>(452)</sup>, 1915; and others. Most of these investigators have arrived at negative results. Theoretically a Hall effect should exist in electrolytes, but the effect is exceedingly small, and the difficulties attendant on its measurement are very great. Layers of the liquid as thin as metal

films cannot be obtained. It seems impossible to eliminate convection currents in the liquid, and the contact differences of potential are large compared with the Hall effect sought.

Bagard (l. c.) is one of the few investigators that claim to have found a true Hall effect in electrolytes. He used solutions of *copper sulphate* and *zinc sulphate*. The layer of liquid was 1.6 mm. thick, and the magnetic field varied from 400 to 1000 gauss. In the copper sulphate solution he claimed to have observed a Hall effect of about  $-0.38 \times 10^{-5}$ . According to Donnan<sup>(247)</sup>, p. 106, the theoretical value of the Hall effect under the conditions employed by Bagard should be of the order of  $-16 \times 10^{-13}$ .

Several of the above investigators, after careful efforts to eliminate all sources of error, failed to detect any Hall effect in electrolytes, and attributed the effect observed by Bagard to the force exerted by the field on the molecules, and to differences in concentration and temperature.

Roiti, Bagard, and Chiavassa employed a capillary electrometer to detect the transverse difference of potential.

The method employed by Moretto (l. c.) in his search for the Hall effect in electrolytes is worthy of mention. He placed strips of filter and tissue paper, saturated with solutions of *sulphuric acid*, *copper sulphate*, *zinc sulphate* and *bismuth nitrate*, between plates of glass. In this way convection currents were probably eliminated. However, he was unable to detect any Hall effect.

Drude and Nernst<sup>(254)</sup>, 1890, looked for the Hall effect in *gelatinized electrolytes*, but found none.

Heilbrun<sup>(254)</sup>, 1903, in seeking for the Hall effect in electrolytes, placed *copper sulphate* between anode and cathode copper plates. A current of 0.023 ampere was sent through the solution for  $1\frac{1}{2}$  hrs., in a field of 7000 gauss. The cathode was divided into upper, middle, and lower sheets. The following table gives the results of deposition of copper when there was no field and when the field was on.

Field	Upper Sheet	Middle Sheet	Lower Sheet
0	0.0124 g.	0.0161 g.	0.0114 g.
7000	0.0132	0.0165	0.0121

The total deposits are respectively 0.0399 g. and 0.0418 g. This difference may be due to a decrease in resistance brought about by a change in temperature. Although the upper plate seemed to have about 1 mg. more deposit when in the field, Heilbrun attributed this to an actual displacement of the salt in solution, and not to a true Hall effect.

Alterthum<sup>(12)</sup>, 1912, endeavored to show that theoretically no Hall effect should exist in electrolytes. His theory is, however, contrary to that of Everdingen, Donnan, Larmor and others.

The Hall effect in electrolytes was carefully investigated by Oxley<sup>(401)</sup>, 1913. He examined the behavior of *copper sulphate*, *silver nitrate*, *cadmium sulphate*, and *copper sulphate gel*.

In previous researches the electrolytic cell had had an area about equal to that of the pole pieces of the magnet, so that the field was not uniform throughout the electrolyte. Voigt<sup>(563)</sup>, 1910, had previously shown that an electrolyte in a non-uniform field will undergo a change in concentration, and thus a difference of potential will be set up. To secure a uniform field throughout the electrolyte, Oxley used a cell and a thin lamina of gel of about 4 cm.<sup>2</sup>, while the area of the pole pieces was 50 cm.<sup>2</sup>

According to Donnan, p. 106, the coefficient of the true Hall effect in an electrolyte is, for complete dissociation, of the order,

$$R = \frac{1}{2} (v - u) = 16 \times 10^{-13}.$$

Then the transverse difference of potential for a fall of 1 volt per cm. will be

$$E = 16 \times 10^{-13} H z,$$

and for a field of 15,000 gauss and  $z = 1$  cm.,

$$E = 2.4 \times 10^{-8} \text{ volt.}$$

Superposed on the true Hall effect, as Donnan (l. c.) has shown, is a difference of potential due to a change in the concentration,

$$E' = \frac{1}{2} H (v + u) \approx \frac{d\pi}{dx}.$$

For a fall of 1 volt per cm.,  $z = 1$  cm., and  $H = 15,000$  gauss,

$$E' = 0.9 \times 10^{-7} \text{ volt.}$$

Thus  $E'$ , which may be called the concentration Hall effect, is larger than the true Hall effect,  $E$ . However, the detection and measurement of  $E'$  is indicative of the existence of a Hall effect.

For copper sulphate solution in a small cell, Oxley found

$$E = 2.17 \times 10^{-8}, \text{ and}$$

$$E' = 8.15 \times 10^{-8}.$$

The latter was the one measured and included the former.

For the copper sulphate gel, he found

$$E = 1.98 \times 10^{-7} \text{ volt, and}$$

$$E' = 7.4 \times 10^{-7} \text{ volt.}$$

In view of all the precautions taken and in view of the fact that the effect reversed with the field, and that the

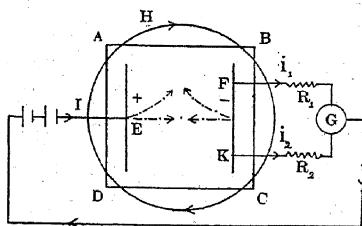


Fig. 66.

observed results are of the calculated order, the author considered that the existence of the Hall effect in electrolytes was established.

In 1915, Righi<sup>(452)</sup> studied the difference of potential set up in an electrolyte by the magnetic field in the following manner. An electrolyte of *copper sulphate* was placed in a shallow horizontal dish,  $ABCD$ , in which were two copper electrodes,  $E$  and  $FK$ , Fig. 66. The primary current  $I$  entered at  $E$  and left the cell at  $F$  and  $K$ . The partial currents,  $i_1$  and  $i_2$ , passed through adjustable resistances,  $R_1$  and  $R_2$ , and entered in opposite directions a differential galvanometer  $G$ , so that no deflection occurred when the magnetic field  $H$  was off.

The following table gives the differences in the partial currents produced by a vertical field of 8,500 gauss.

$I$	$(i_2 - i_1)$	$(i_2 - i_1) \div I$
0.005 amp.	0.000017 amp.	0.0037
0.010	0.000050	0.0050
0.020	0.000170	0.0085
0.030	0.000350	0.0117

This difference between the currents  $i_1$  and  $i_2$  is probably not a simple Hall effect. The magnetic field causes a deviation of the positive and negative ions both in the same direction, as indicated in Fig. 66. The region near  $F$  becomes more concentrated, and a difference of potential results. The difference in the partial currents,  $i_1$  and  $i_2$ , may be considered as a concentration Hall effect plus a true Hall effect.

### 3. Theories of the Hall Effect in Electrolytes

#### a. Donnan's Theory of the Hall Effect in Electrolytes

Donnan<sup>(147)</sup>, 1898, supposed the primary current in the electrolyte to flow under an E.M.F. of  $\frac{d\pi}{dx}$ , in the direction of the axis of  $x$ , that the lines of magnetic force lay along the axis of  $y$ , and that the Hall difference of potential,  $\frac{de}{dz}$ , was along the axis of  $z$ .

For a concentration hardly appreciable he deduced this expression for the Hall E.M.F.,

$$\frac{de}{dz} = we(v-u)H \frac{d\pi}{dx}.$$

For complete dissociation the expression is

$$\frac{de}{dz} = \frac{1}{2} we(v-u)H \frac{d\pi}{dx},$$

in which  $w$  is the valency of each ion,  $e$  the quantity of electricity traveling per gram-equivalent of ionic matter,  $u$  and  $v$  the respective velocities of the positive and negative ions under unit potential gradient.



These equations are in reality the same as those of Van Everdingen (p. 106), and in each case for complete dissociation, the Hall coefficient is given by the relation,

$$R = \frac{1}{2} (v - u).$$

Donnan showed that the ratio of the Hall E. M. F. to the primary E. M. F. is of the order of  $16 \times 10^{-13}$ : 1, for a field between 400 and 1000 gauss. He suggested that the effect might become measurable in a very strong magnetic field, if the fall of potential in the plate were 10,000 volts per cm. Since fields of 30,000 to 50,000 gauss may now be obtained with the improved semi-circular electromagnet, and since there are large storage batteries now in existence with 20,000 to 30,000 elements, it would seem possible to measure the Hall effect in electrolytes in a very dilute state.

*b. Van Everdingen's Theory of the Hall Effect in Electrolytes*

Van Everdingen<sup>(182)</sup>, 1898, deduced an expression for the Hall effect in electrolytes, in accordance with the following assumptions: The primary current in the electrolyte flows under an impressed E. M. F.,  $E_x$ , along the axis of  $X$ . The velocities of the positive and negative ions under a unit difference of potential are respectively  $u$  and  $v$ . The magnetic field,  $H$ , applied in the direction of the axis of  $Y$ , sets up a difference of potential  $E_z$ , along the axis of  $Z$ .

The Hall difference of potential for a very slightly dissociated salt is then,

$$E_z = -E_x H(u - v), \text{ and } \frac{dc_2}{dx} = 0,$$

where  $c_2$  is the concentration of the salt.

For complete dissociation,

$$E_z = -\frac{1}{2} E_x H(u - v), \text{ and } \frac{1}{c_1} \cdot \frac{dc_1}{dx} = E_x H \frac{(u + v)}{2A},$$

in which  $c_1$  is the concentration of both positive and negative ions, and  $A$  a constant related to the constant of diffusion for complete dissociation of the electrolyte.

*c. Larmor's Theory of the Hall Effect in Electrolytes*

Larmor <sup>(310)</sup>, 1900, developed an expression for the Hall effect in electrolytes, for a dilute solution, in which the ionization is complete. The transverse electromotive force set up by a magnetic field,  $H$ , in a electrolyte carrying a current  $I$ , he found to be

$$E = \frac{v-u}{v+u} \cdot \frac{IH}{2ne},$$

in which  $v$  and  $u$  are the velocities of drift respectively of the cations and anions under a unit electric force,  $n$  the coefficient of diffusion of the ions, and  $e$  is the ionic charge.

Comparing the above expression with the ordinary Hall effect equation,

$$E = \frac{RHI}{d},$$

and putting  $d=1$ , we have for the Hall coefficient,

$$R = \frac{v-u}{v+u} \cdot \frac{1}{2ne}.$$

Since  $v-u$  is small compared with  $v+u$ , it is evident that the Hall effect in electrolytes must be small. The author pointed out that, if the primary current flows from plate to plate in the electrolyte, so that the stream lines are parallel, the resistance of the electrolyte will not be changed by the magnetic field, but if the current flows between point electrodes the resistance will be affected.

*d. Wind's Equation for the Hall Effect in Electrolytes*

Wind <sup>(590)</sup>, 1894, deduced the following equation for the Hall effect in electrolytes:

$$R = \frac{1}{ne} \cdot \frac{v-u}{v+u}.$$

This equation is double the magnitude of that of Larmor above. This difference is probably due to the fact that Wind did not consider the dissociation of the electrolyte to be complete, while Larmor did so consider it.

## CHAPTER VII

## HALL EFFECT IN GASES

## 1. Hall Effect in Discharge Tubes

Boltzmann <sup>(86)</sup> in 1886 investigated the Hall effect in rarified gases in a thin flat Geissler tube, Fig. 67, placed between the poles of an electromagnet. The secondary electrodes were platinum wires, *S, S*, sealed in the sides of the tube. The pressure in the tube was from 2 to 5 mm. of mercury. The Hall effect in *air* was found to be negative, while that in *hydrogen* and *carbon dioxide* was positive. In a

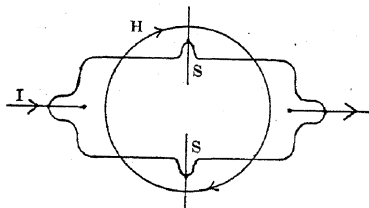


Fig. 67.

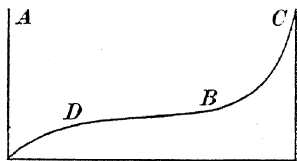


Fig. 68.

field of 1800 gauss the secondary current was  $1/60$  to  $1/30$  that of the primary current *I*.

Boltzmann also employed the Righi method (p. 18), by allowing the primary current to enter the tube by one electrode and to leave by two electrodes connected to a differential galvanometer.

Accurate quantitative determination of the Hall effect in gases is beset by many difficulties. The fall in potential in gases in a flame, or in a discharge tube, occurs almost entirely near the electrodes, as shown in Fig. 68. The fall at the cathode, *C*, is very rapid, a small drop occurs near the anode, *A*, while along the intervening space, *DB*, the fall is exceedingly small. The Hall electrodes are usually placed in the region *DB*, equidistant from the primary electrodes. Since the fall in potential in this region is small, and since the Hall effect depends on the fall in potential, the magnitude of the effect must be small.

The Hall effect in gases at low pressure has been determined by Wilson <sup>(594)</sup>, 1901. The apparatus employed, Fig. 69, consisted of a glass tube, *AB*, with primary aluminium

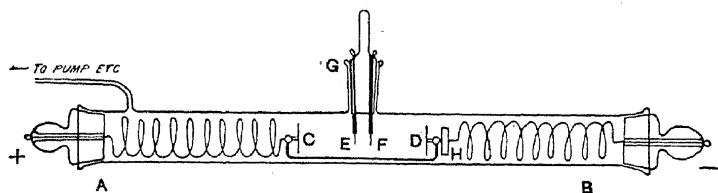


Fig. 69.

electrodes, *C*, *D*, which could be moved along the tube by means of a magnet, and a ground glass stopper, *G*, into which were sealed the secondary electrodes, *E*, *F*. The secondary electrodes could be rotated into an equipotential surface. The Hall difference of potential was measured by means of a quadrant electrometer.

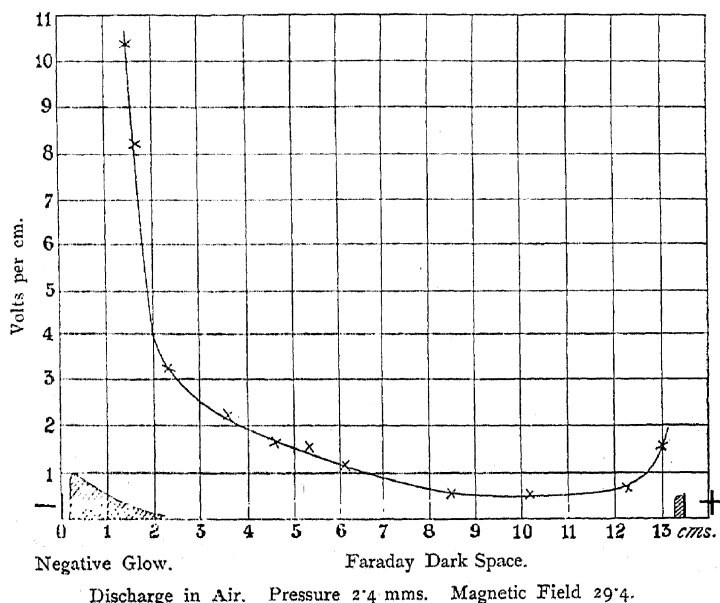


Fig. 70.

In general, the Hall effect in gases at low pressure was found to be proportional to the potential gradient in the positive column and to the magnetic field, but independent of the main discharge current through the tube.

The variation of the Hall effect in *air* at low pressure along the discharge tube is shown in Fig. 70. The effect is greatest near the cathode where the fall in potential is most rapid, reaches a minimum near the Faraday dark space, and increases again as the anode is approached.

This variation is in accord with the fall in potential in Fig. 68.

The Hall effect was found by Wilson to vary inversely with the pressure, as shown in the table below.

*Hall Effect in Air at Low Pressure*

Pressure, $p$ , in mm.	Hall E.M.F., $Z$ , in Volts/cm.	Magnetic Field, $H$	$\frac{Zp}{H}$
2.91	0.356	45.2	$2.29 \times 10^{-2}$
2.23	0.475	45.8	2.32
1.85	0.597	46.3	2.38
1.22	0.915	44.7	2.49
0.983	1.20	46.3	2.54
0.650	1.85	46.8	2.56
0.508	2.42	46.3	2.76
0.333	3.24	46.8	2.31
0.266	4.60	46.4	2.64
Mean:			$2.48 \times 10^{-2}$

According to the above, the transverse difference of potential for *air* is

$$Z = 0.0248 \frac{H}{p}.$$

For *hydrogen*, Wilson found  $Z = 0.0205 \frac{H}{p}$ ; for *oxygen*,  $Z = 0.00379 \frac{H}{p}$ .

The Hall coefficient is given by the relation:

$$R = \frac{Z}{XH}.$$

The potential gradient along the uniform portion of the tube,  $D-B$ , Fig. 68, was found to vary as the square root of the pressure. For *air*,  $X=34.9\sqrt{p}$ ; for *hydrogen*,  $X=28\sqrt{p}$ ; for *oxygen*,  $X=26.9\sqrt{p}$ .

The above values of  $Z$  and  $X$  were found to hold for pressures from 0.1 mm. to 3 mm.

Wilson<sup>(594)</sup> showed that, if the ionization and recombination are neglected, the equation for the Hall effect in rarefied gases is identical with that for a completely dissociated salt solution, and that

$$Z = \frac{1}{2} H X (v - u),$$

where  $v$  and  $u$  are respectively the velocities of the negative and positive ions per one volt per cm. According to the above equation, the Hall coefficient is,  $R = \frac{1}{2} (v - u)$ .

Using the above relations for  $X$ ,  $Z$ ,  $H$ ,  $p$ ,  $v$ ,  $u$ , Wilson and Martyn<sup>(598)</sup>, 1907, computed values for  $v$  and  $u$  for *air* and *hydrogen*. The results are given in the table below.

Gas	$\frac{Zp}{H}$	$p$	$Xp$ volts/cm.	$v$ in cm. per sec. per volt/cm.	$u$ in cm. per sec. per volt/cm.
Air . . . .	0.0248	3.8 mm.	68	19,700	450
Hydrogen	0.0205	„	55	25,000	5,000

Wilson and Martyn discovered that a definite relation existed between the Hall effect in rarefied gases and the rotation of the electric discharge in a radial magnetic field. The electric discharge was made to rotate continuously round a pole of a magnet inserted in the discharge tube. It was found that in rarefied *air*, *hydrogen*, and *nitrogen*, the transverse velocity,  $V$ , of the discharge was proportional to the pressure,  $p$ . Hence

$$V = C \frac{H}{p}$$

in which  $C$  is a constant depending on the gas. But the Hall difference of potential is given by the relation

$$Z = D \frac{H}{p}.$$

Dividing, we obtain

$$Z = \frac{D}{C} V.$$

Which means that the Hall effect is proportional to the transverse velocity of the discharge.

The Hall effect in a *mercury vapor lamp* was determined by Schenkel <sup>(471)</sup>, 1906. The Hall effect is given by the equation

$$R = Z \div XH = \frac{1}{2} (K_2 - K_1),$$

in which  $Z$  is the transverse difference of potential set up in the tube by the field  $H$ ,  $X$  is the fall in potential along the tube, and  $K_1$  and  $K_2$  are respectively the mobilities of the positive and negative ions. The table below gives the results.

*Hall Effect in a Mercury Vapor Tube*

$H$	Temp. Low	Temp. High	Pressure Min.	Pressure Max.	$K_2 - K_1$ Min.	$K_2 - K_1$ Max.
0.60 gauss	76.5° C.	77.5° C.	0.31 mm.	0.32 mm.	$1.96 \times 10^7$	$2.02 \times 10^7$
1.20	67.5	76.5	0.21	0.31	1.60	2.35
1.80	63.5	73.0	0.18	0.26	1.38	2.0
2.40	70.0	77.5	0.24	0.32	1.46	1.84
3.00	69.0	78.0	0.24	0.32	1.15	1.58

According to the above, the mean difference in velocity of the negative and positive ions is  $(K_2 - K_1) p^{\text{mm. of Hg}} 1.7 \times 10^7$  cm. per sec. per volt.

This difference in mobility of mercury ions is more than 100 times that found by Wilson for air, oxygen, and hydrogen. This large excess of the negative mobility probably accounts for the unipolar conductivity of the mercury vapor lamp.

## 2. Hall Effect in Flames

The Hall effect in flames was investigated by Marx <sup>(355)</sup>, 1900, in the following manner. A specially devised flat Bunsen burner,  $B$ , Fig. 71, was used. The gas pressure was kept practically constant by means of an automatic pressure gauge. Salt solutions of varying strengths were sprayed into

the flame by means of a Gouy vaporizer. The primary electrodes,  $P, P$ , were of platinum gauze, the mesh of the cathode,  $C$ , was coarser than that of the anode,  $A$ . The secondary electrodes,  $S, S$ , were of fine platinum wire. The flame required so much space between the poles of the electromagnet, that only fields from 3000 to 8500 gauss could be obtained. The primary electrodes were charged from a battery of 72 Clark cells, and the potential of the cathode was kept at about five times the potential of the anode. The potential of the primary electrodes was measured by means of a quadrant electrometer, while the Hall E.M.F. at the secondary electrodes was measured by a Dolezalek-Nernst electrometer.

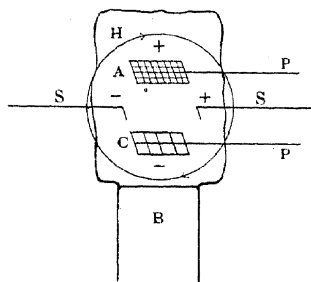


Fig. 71.

Marx deduced the following expression for the Hall coefficient:

$$R = \left( \frac{e}{z} \div \frac{\pi}{x} \right) \cdot \frac{I}{H} = \frac{I}{2} (v - u),$$

in which  $e$  is the transverse Hall E.M.F.,  $z$  the distance between the Hall electrodes, and  $\frac{\pi}{x}$  the potential gradient along the primary current, and  $v$  and  $u$  the velocities of the ions.

The following table gives the Hall effect in a flame sprayed with a  $2 \times$  normal solution of  $KCl$ .

Hall Effect in  $2 \times$  Normal  $KCl$  Flame

Air pressure 104 cm. Primary electrodes at $+17.2$ and $-90.4$ volts				
$e$	$z$	$\frac{\pi}{x}$	$H$	$R$
0.0706 volt	1.48 cm.	2.77 volts/cm.	5075	$-3.81 \times 10^{-6}$
0.1386	1.60	6.36	3600	$-3.78$
0.2987	1.73	5.8	8180	$-3.64$
0.4601	1.66	8.5	8400	$-3.88$



According to the above results, the Hall effect is independent of the field, the potential gradient, and the distance between the secondary electrodes.

The next table gives the variation of the Hall effect with concentration of the salt solution.

*Hall Effect in KCl Flames*

	0-normal	$\frac{1}{8}$ normal	$\frac{1}{4}$ normal	$\frac{1}{2}$ normal	$2\times$ normal	$3.8\times$ normal
$R\times 10^6$	-10.18	-8.24	-5.40	-4.26	-3.78	-3.75

For a saturated flame,  $R = -3.7 \times 10^{-6}$ .

The curve in Fig. 72 shows that the Hall coefficient,  $R$ , falls rapidly as the concentration increases from 0 to  $\frac{1}{2}\times$ normal solution, and then approaches a constant value.

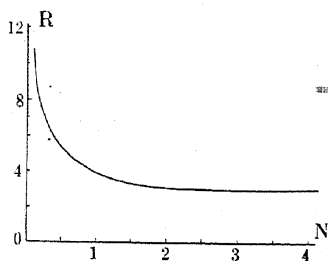


Fig. 72.

The table below gives the Hall effect as determined by Marx for flames sprayed with  $2\times$ normal solution of several univalent salts.

*Hall Effect in  $2\times$ Normal Salt Flames*

Solution	At. Wt.	$R$	$\sqrt{\text{At. Wt.}} \times R$
CsCl . . . .	Cs, 132.9	$-1.72 \times 10^{-6}$	19.83
RbCl . . . .	Rb, 85.4	-2.7	24.95
KCl . . . .	K, 39.14	-3.78	23.64
NaCl . . . .	Na, 23.05	-5.06	24.3
LiCl . . . .	Li, 7.03	-7.86	20.79

The Hall effect increases while the atomic weight decreases, and the product of the square root of the atomic weight by  $R$  is nearly constant.

Marx found that if the cathode and anode were kept at the same temperature, the fall in potential occurred almost entirely near the cathode. If, however, the cathode was kept

considerably hotter than the anode, the fall in potential was more evenly distributed between the electrodes. For this reason the mesh of the cathode was made coarser than that of the anode, and the anode was always placed in the upper and cooler portion of the flame. Hence the primary current could not be reversed.

The sign of the Hall effect in all the salts examined by Marx was negative.

Moreau<sup>(377)</sup>, 1902, measured the Hall effect in flames impregnated with *potassium* and with *sodium chloride* solutions. He measured the mobilities of the positive and negative ions,  $K_1$  and  $K_2$ , and computed the Hall coefficient,  $R$ , from the relation  $R = (K_1 - K_2)$ . In this, as in the Wilson formula below, the dissociation is incomplete, while in the equation of Marx the dissociation is considered complete.

The following table gives Moreau's calculated and Marx's observed values for  $R$  for flames sprayed with  $KCl$  solutions.

*Hall Effect in KCl Flames*

	Pure Flame	$\frac{1}{8}$ normal	$\frac{1}{4}$ normal	$\frac{1}{2}$ normal	$2\times$ normal
$R > 10^6$ Calc.	— 12.7	— 8.2	— 7	— 6.2	— 5.2
$R > 10^6$ Obs.	— 10.2	— 8.24	— 5.4	— 4.26	— 3.8

Wilson<sup>(507)</sup>, 1914, pointed out that, since in a Bunsen flame the velocity of the negative ions is large compared with the velocity of the positive ions,  $K_1$  may be neglected, and  $X \div HY = K_1 - K_2$  becomes  $K_2 = -X \div HY$ . The negative ions in a flame are supposed to be free electrons, hence  $K_2$  should be independent of the nature and concentration of the salt in the flame.

Wilson suggested that Marx's results, which seem contrary to theory, were due to the fact that he used a small flat flame, that the primary electrodes were placed in a vertical direction and 3 cm. apart, and that the secondary electrodes were the same distance apart in a horizontal direction. The primary

electrodes (Fig. 71, p. 113) are equipotential surfaces, and their nearness would tend to prevent a difference of potential being set up in the Hall electrodes. The greater concentration of the salt and hence the greater conductivity of the flame

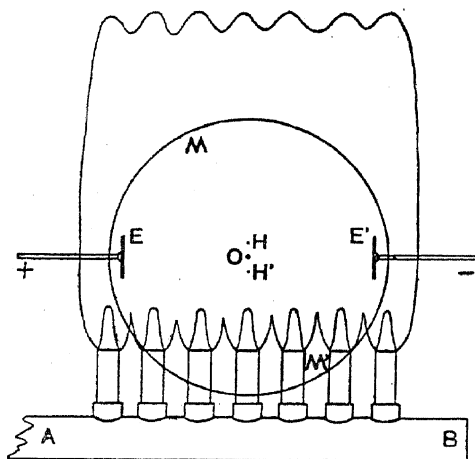


Fig. 73.

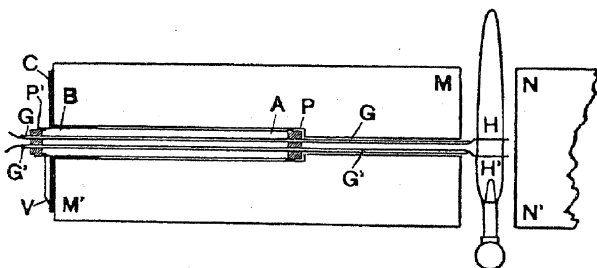


Fig. 74.

would increase the influence of the primary electrodes on the secondary electrodes, and in consequence of this masking, the Hall effect would seem to diminish with concentration.

To avoid this influence of the primary on the secondary electrodes, Wilson used a large thin flame  $12 \times 12 \times 1.5$  cm., Figs. 73, 74. The electrodes  $E$  and  $E'$  were platinum disks 5 cm in diameter and were kept 9 cm. apart. The Hall

effect was measured by two fine platinum wires,  $H$  and  $H'$ , placed near each other, and connected to a quadrant electrometer. When the field  $M$  was on, the plane of the wires  $H$  and  $H'$  was rotated until they were in an equipotential plane. Then the angle of rotation of the equipotential plane by the magnetic field is given by the equation

$$\tan \Theta = \frac{X}{Y},$$

in which  $X$  and  $Y$  are respectively the potential gradients along the axes  $x$  and  $y$ . For a constant field of 7,700 gauss, Wilson obtained the following results.

Hall Effect angle $\Theta$	LiCl 2%	Na <sub>2</sub> CO <sub>3</sub> 2.4%	K <sub>2</sub> CO <sub>3</sub> 2.6%	RbCl 2%
	10.9°	10.7°	10.7°	10.8°

Accordingly, it seems that the Hall effect and the velocity of the negative ions in alkali metal salts are independent of the metal and its atomic weight.

The following result was obtained for a change in concentration of  $K_2CO_3$ , in a field of 7,700 gauss.

Concentration	10%	2.6%	0.26%
$\Theta$	11.0°	10.7°	10.0°

Marx found a decided decrease in the Hall effect with increase in concentration, while here we have a slight increase. For a constant concentration the Hall angle decreases with the field as shown below.

Salt	Concentration	Field	$\Theta$
K <sub>2</sub> CO <sub>3</sub>	2.6%	7,700	10.7°
"	"	4,800	7.0°
"	"	2,250	3.1°

Here the  $\tan \Theta$  varies nearly directly as the field strength.

### 3. Hall Effect in the Electric Arc

The Hall effect in the electric arc was investigated by Child<sup>(117)</sup>, 1904. The secondary electrodes were carbon pencils,

placed so that these points were at the same potential. It was found that the magnetic field produced a difference of potential of about 1.5 volts. This effect could not be produced by blowing the arc to one side, nor by shifting the electrodes to one side of the arc. The effect was found to diminish with decreasing field strength. Change in the strength of the current through the arc and change in the length of the arc did not appreciably influence the magnitude of the effect.

The effect was found to be practically the same in an enclosed arc. Reduced pressure produced no marked change in the effect, until a pressure of 20 cm. of mercury was reached. Between pressures of 20 cm. and 10 cm., the effect diminished rapidly, and at a pressure of 1 cm., the effect disappeared entirely. The effect seemed independent of the anode and cathode drop, but decreased with the total difference of potential in the arc, and became inappreciable when the voltage across the arc was less than 30 volts.

Salts sprayed into the arc diminished the effect, and potassium nitrate was found to reduce the effect almost to zero.

If the large difference of potential set up by the magnetic field in the electric arc is a true Hall effect, the negative ions must have a very great velocity. This may be accounted for by the large fall in potential along the arc and the high temperature of the arc.

Child further observed that the Hall effect in the electric arc is zero, when the fall in potential at the anode is small. This he considered due to the positive ions having a greater velocity than the negative ions.

The Hall effect in the electric arc seems worthy of further investigation.

#### 4. Thomson's Theory of the Hall Effect in Gases

The following theory of the Hall effect in gases, developed by Sir J. J. Thomson <sup>(524)</sup>, applies to all cases of conduction

of electricity through gases containing ions, whether in flames or discharge tubes.

The primary current is supposed to flow parallel to the axis of  $x$ , and the magnetic field acts at right angles to this direction along the axis of  $z$ . The components of the electric force parallel to  $x$  and  $z$  are respectively  $X$  and  $Z$ . The velocities of the positive and negative ions under unit electric force are  $u$  and  $v$ , while  $m$  and  $n$  are the number of positive and negative ions respectively per c.c. at any point. It is assumed that the ions behave as a perfect gas.

Under a magnetic field of strength,  $H$ , it is shown that the transverse E.M.F. is given by the equation

$$Z = \frac{XH(nv - mu)}{m + n}.$$

The current carried by the positive ions is  $i_p = emuX$ , and that carried by the negative ions is  $i_n = envX$ . Hence the above equation may be written,

$$Z = \frac{H(i_n - i_p)}{e(m + n)}.$$

At a place where there is no free electricity,  $m = n$ , and the equation becomes

$$Z = \frac{1}{2} XH(v - u),$$

and the Hall coefficient is

$$R = \frac{Z}{XH} = \frac{1}{2} (v - u).$$

The above equation is the same as that deduced by Van Everdingen<sup>(181)</sup>, Donnan<sup>(147)</sup>, and Larmor<sup>(310)</sup> for the Hall effect in electrolytes, by Marx<sup>(355)</sup> for the effect in flames, and by Wilson<sup>(594)</sup> for the effect in discharge tubes.

Wilson<sup>(594)</sup> has pointed out, that, in the case of electric discharge in tubes, the magnetic field alters the distribution of the current and ionization, so that a concentration gradient is set up across the tube, and that consequently  $Z$  in rarefied gases is only one-half what it is in flames, in which the

magnetic field leaves the ionization unchanged. Hence the Hall effect in flames is given by the equation

$$R = \frac{Z}{XH} = (v-u).$$

This relation only holds true in portions of the flame a little way in from the edges where  $Z$  and  $X$  are uniform, that is, where ionization and recombination are equal.

Further investigation of the action of a magnetic field on cathode and positive rays, on ionic clusters in discharge tubes, on flames, on electric arcs, and other electric discharges in gases, may shed valuable light on the influence of the magnetic field on electrons and atoms in metals, and thereby contribute to a clearer understanding of the Hall and allied phenomena.

#### Hall Effect in the Elements

Element	Temperature	$H$	$R$	Observer	
Aluminium . . .	20° C.	Ind. Field	— 0·00038	Ettingshausen and Nernst	
	—	—	— 0·00040	Zahn and Schmidt	
	20° Abs.	—	— 0·000248	Alterthum	
	83° "	—	— 0·000200	"	
	194° "	—	— 0·000390	"	
	291° "	—	— 0·000390	"	
Antimony . . . .	Room temp.	—	+ 0·114	Hall	
	20° C.	4,080	+ 0·192	Ettingshausen and Nernst	
	C. P.	—189°	6,160	+ 0·246	Barlow
	"	21·5°	1,750	+ 0·219	"
	"	"	24,700	+ 0·186	"
	"	58°	6,160	+ 0·202	"
	"	18°	Ind. Field	+ 0·219	Zahn
	"	20° Abs.	—	+ 0·152	Alterthum
	"	83° "	—	+ 0·163	"
	"	194° "	—	+ 0·140	"
	"	291° "	—	+ 0·125	"
	Bismuth . . . .	Room temp.	—	— 8·58	Hall
20° C.		1,650	— 10·27	Ettingshausen and Nernst	
"		11,100	— 4·95	"	
C. P.		18° C.	3,930	— 6·33	Zahn
"		"	10,570	— 4·82	"

Element	Temperature	$H$	$R$	Observer
Bismuth . C. P.	14.5°	890	— 5.78	Traubenberg
"	10.4°	"	— 2.83	"
"	257°	"	— 1.113	"
"	270° (melting)	"	— 0.25	"
	4.25° Abs.	1,000	— 86.3	Onnes and Hof
	"	11,300	— 85.7	"
Cadmium . . . .	20° C.	12,370	+ 0.00055	Ettingshausen and Nernst
	16°	—	+ 0.00088	Unwin
	20° Abs.	Ind. Field	+ 0.0020	Alterthum
	83° "	"	+ 0.000795	"
	194° "	"	+ 0.000605	"
	291° "	"	+ 0.000589	"
	14.5° "	11,800	+ 0.00191	Onnes and Hof
	20.5° "	"	+ 0.00176	"
	200° "	12,000	+ 0.00055	"
Carbon . . . . . (Microphone disc)	20° C.	11,160	— 0.176	Ettingshausen and Nernst
	90°	10,270	— 0.160	Zahn
Graphite . . .	14.5° Abs.	11,800	+ 1.52	Onnes and Hof
	20.5° "	"	+ 2.22	"
	200° "	"	+ 0.74	"
	17° C.	5,460	$R_a$ — 0.71	Gottstein
	"	"	$R_i$ — 0.487	"
	00.2°	"	$R_a$ — 0.622	"
	"	"	$R_i$ — 0.441	"
Cerium . . . . .	24°	Ind. Field	+ 0.00192	Smith, A.W.
Cobalt . . . . .	Room temp.	3,463	+ 0.00246	Hall
	"	—	+ 0.00245	Unwin
" Electrolytic	24° C.	4,400	+ 0.00161	Zahn
" I. . . . .	Room temp.	5,650	+ 0.00155	Ettingshausen and Nernst
II. . . . .	"	"	+ 0.00459	"
	24° C.	17,000	+ 0.0024	Smith, A.W.
Copper . . . . .	24°	Ind. Field	— 0.00052	Hall
	Room temp.	12,010	— 0.00052	Ettingshausen and Nernst
	"	—	— 0.00055	Unwin
" I. . . . .	20° C.	Ind. Field	— 0.00054	Zahn
II. . . . .	"	"	— 0.000428	"
	20° Abs.	"	— 0.000605	Alterthum
	83° "	"	— 0.000792	"
	194° "	"	— 0.000553	"



Element	Temperature	<i>H</i>	<i>R</i>	Observer	
Copper . . . . .	291° Abs.	Ind. Field	— 0'000528	Alterthum	
	14'5° "	"	— 0'000656	Onnes and Beckman	
	20'3° "	"	— 0'000662	"	
	290° "	"	— 0'000492	"	
Gold . . . . .	24° C.	"	— 0'00066	Hall	
	Room temp.	9,010	— 0'00071	Ettingshausen and Nernst	
	20° Abs.	Ind. Field	— 0'000857	Alterthum	
	83° "	"	— 0'000726	"	
	194° "	"	— 0'000712	"	
	291° "	"	— 0'000704	"	
	130° C.	4,320	— 0'000769	Frey	
	840°	4,790	— 0'000750	"	
	14'5° Abs.	Ind. Field	— 0'000982	Onnes and Beckman	
	20'3° "	"	— 0'000981	"	
Indium . . . . .	290° Abs.	"	— 0'000724	"	
	24° C.	"	— 0'000073	Smith, A. W.	
Iridium I. . . . .	20°	"	+ 0'000402	Zahn	
	II. . . . .	"	+ 0'000364	"	
Iron . . . . .	Room temp.	—	+ 0'00785	Hall	
	"	5,750	+ 0'0113	Ettingshausen and Nernst	
" Soft . . . . .	"	—	+ 0'0087	Unwin	
	12'9° C.	5,300	+ 0'00794	Hall and Campbel	
" Silesia . . . . .	18°	6,290	+ 0'0108	Zahn	
	24°	17,000	+ 0'0110	Smith, A. W.	
" Soft . . . . .	18°	3,700	+ 0'0114	Moreau	
	" . . . . .	37°	17,000	+ 0'0103	"
" I. . . . .	20° Abs.	—	+ 0'00144	Alterthum	
	" . . . . .	83° "	—	+ 0'00133	"
" . . . . .	" . . . . .	194° "	—	+ 0'00409	"
	" . . . . .	291° "	—	+ 0'01123	"
Lead . . . . .	20° C.	11,400	+ 0'00009	Ettingshausen and Nernst	
	4'25° Abs.	5,000	+ 0'00008	Onnes and Hof	
	"	11,300	+ 0'00018	"	
	2'8° "	"	+ 0'00013	"	
Lithium . . . . .	24° C.	Ind. Field	— 0'00170	Smith, A. W.	
	Magnesium . . . . .	Room temp.	"	— 0'0035	Hall
20° C.		7,390	— 0'00094	Ettingshausen and Nernst	

Element	Temperature	$H$	$R$	Observer
Manganese . . .	—	—	— 0'00093	Zahn and Schmidt
Molybdenum . .	24° C.	Ind. Field	+ 0'00126	Smith, A. W.
Nickel . . . . .	Room temp.	—	— 0'0147	Hall
	"	1,550	— 0'0242	Ettingshausen and Nernst
	"	—	— 0'00294	Unwin
	18° C.	6,290	— 0'0125	Zahn
„ Electrolytic	"	10,620	— 0'00469	"
„ Pure . . .	37°	1,700	— 0'0123	Moreau
„ . . . .	"	13,800	— 0'00508	"
„ I. . . . .	20° Abs.	—	— 0'00152	Alterthum
„	83° „	—	— 0'00138	"
„	194° „	—	— 0'00280	"
„	291° „	—	— 0'00470	"
	4'25° „	11,300	< 0'00001	Onnes and Hof
	2'8° „	"	< 0'00001	"
Palladium . . .	20° C.	5,800	— 0'00115	Ettingshausen and Nernst
„ I. . .	"	Ind. Field	— 0'000691	Zahn
„ II. . .	"	"	— 0'001112	"
	Room temp.	20,300	— 0'000769	Smith, A. W.
Saturated with hydrogen . .	"	"	— 0'000754	"
	14'5° Abs.	Ind. Field	— 0'001374	Onnes and Beckman
	20'3° „	"	— 0'001368	"
	290° „	"	— 0'000675	"
Platinum . . . .	Room temp.	—	— 0'00024	Hall
	20° C.	12,000	— 0'00024	Ettingshausen and Nernst
	"	Ind. Field	— 0'000127	Zahn
	20° Abs.	"	— 0'000199	Alterthum
	83° „	"	— 0'000198	"
	194° „	"	— 0'0000904	"
	291° „	"	— 0'000202	"
	160° C.	5,500	— 0'000188	Frey
	1325°	5,300	— 0'000271	"
Selenium . . . .	—	—	Negative	Amaduzzi
Silicon . . . . .	23° C.	Ind. Field	+ 41'00	Smith, A. W.
„ I. . . .	50°	"	+ 23'00	Buckley
„ II. . . .	"	"	— 39'00	"
„ III. . . .	"	"	+ 12'00	"
	18'7°	6,150	$R_a$ + 206	Gottstein

Element	Temperature	$H$	$R$	Observer
Silicon . . . . .	18.7° C.	6,150	$R_i + 120$	Gottstein
	89.7°	"	$R_a + 125$	"
	"	"	$R_i + 87$	"
Silver . . . . .	Room temp.	—	— 0.00086	Hall
	20° C.	9,160	— 0.00083	Ettingshausen and Nernst
	Room temp.	—	— 0.000838	Unwin
	20° C.	Ind. Field	— 0.000897	Zahn
	2.8° Abs.	11,300	— 0.0016	Onnes and Ho
	4.25° "	"	— 0.0016	"
	14.5° "	Ind. Field	— 0.0009	Onnes and Beckman
	20.3° "	"	— 0.001014	"
	290° "	"	— 0.00080	"
	20° C.	8,700	— 0.0025	Ettingshausen and Nernst
Steel . . . . .	20° C.	—	+ 0.0175	"
Tantalum . . .	24°	Ind. Field	+ 0.00101	Smith, A. W.
Tellurium . . .	20° C.	2,800	+ 530	Ettingshausen and Nernst
	27.1°	Ind. Field	+ 784	Wold
	65°	4,000	+ 430	Lloyd
	—	—	+ 500	Zahn
	21.8°	8,800	+ 536	Smith, A. W.
" I. . .	20.3°	"	+ 621	"
" II. . .	24°	Ind. Field	+ 0.00024	"
Thallium . . . .	Room temp.	—	— 0.00002	Hall
Tin . . . . .	"	4,970	— 0.00004	Ettingshausen and Nernst
	2.8° Abs.	11,300	+ 0.000098	Onnes and Hof
	4.25° "	"	+ 0.000026	"
	24° C.	Ind. Field	+ 0.00118	Smith, A. W.
Tungsten . . . .	Room temp.	—	+ 0.00082	Hall
Zinc . . . . .	"	8,550	+ 0.00041	Ettingshausen and Nernst
	"	—	+ 0.000763	Unwin
	20° C.	5,000 to 11,000	+ 0.00104	Zahn
	22°	25,000	+ 0.00033	Barlow
	80°	4,330	+ 0.000530	Frey
	200°	5,120	+ 0.000446	"
	400°	5,400	+ 0.000534	"

## CHAPTER VIII

## 1. The Corbino Effect

Corbino <sup>(120)</sup>, 1911, discovered three galvanomagnetic effects, which are brought into existence under the following conditions:

1. When a circular metallic disk, in which flows a uniform radial electric current, is placed in a magnetic field perpendicular to the lines of force, a circular current is set up in the disk. The density of the secondary current is inversely proportional to the radius of the disk.

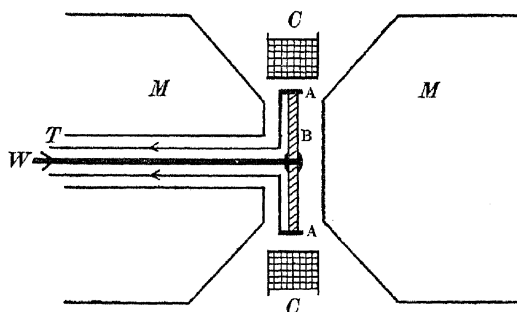


Fig. 75.

2. If the disk carrying the radial current is placed in the magnetic field so that the plane of the disk is oblique to the lines of force, the field produces in the disk a torque that tends to turn the disk.

3. If the disk, carrying no current, is placed at right angles to the lines of force, a radial current is induced in the disk when the magnetic field is excited, and an opposite current is set up when the field is removed.

In his investigation of the above effects, Corbino employed a disk of *bismuth*, *B*, Fig. 75, 6.5 cm. in diameter and 0.28 cm. thick. In the centre of the disk was inserted a stout copper wire, *W*. Around the edge of the disk was a copper ring, *AA*, to which was fastened a copper tube, *T*. The radial

current entered the disk through the wire  $W$ , and left by the tube,  $T$ . The tube  $T$  enclosing the wire  $W$  was inserted in one pole piece of the electromagnet,  $MM$ . The hollow coil,  $CC$ , was connected to a galvanometer. Whenever the radial current in the disk was made or broken, in the presence of the magnetic field, a current was induced in the coil  $CC$  by the circular current set up in the disk by the action of the field. It was found that the circular current changed direction with that of the primary radial current and of the magnetic field. In general the circular current varied directly as the strength of the radial current and as the magnetic field.

With a radial current of about 15 amperes, Corbino obtained the following results for bismuth.

*Corbino Effect in Bismuth*

Field, $H$	Current $C$ , in Coil $CC$	$\frac{C}{H} \times 10^5$
3700	0.26 amp.	7
5700	0.31	5.6
7000	0.34	4.9
8200	0.36	4.3
9000	0.37	4

The Corbino effect in bismuth is thus seen to decrease in higher fields.

Corbino developed expressions for the several effects discovered by him, based on the supposition that the electric current is carried by both positive and negative ions.

For the first effect noted above, he obtained for the circular current,  $C$ , the equation:

$$C = -\frac{EIH}{2\pi} \log \frac{r_1}{r_2},$$

in which  $E$  is the differential moment of the ions and equal to

$$\frac{e\sigma_1 v_1 - e\sigma_2 v_2}{\sigma},$$

where  $e$  is the electric charge;  $v_1$  and  $v_2$  the velocities of the

positive and negative ions;  $\frac{\sigma_1}{\sigma}$  and  $\frac{\sigma_2}{\sigma}$  the partial conductivities of the ions;  $I$  the primary radial current;  $H$  the field;  $r_2$  and  $r_1$  the external and internal radii of the disk. Since  $e$  is negative,  $Q$  is positive when the mobility of the positive ions is greater than that of the negative.

The Corbino effect is said to be positive when, while the primary radial current is flowing toward the centre of the disk, the circular current set up in the disk has the same direction as the magnetizing current in the coils of the electromagnet. The effect in *bismuth* was found to be *negative*, and that in *antimony* *positive*.

The expression found by Corbino for the torque set up in the plate by the magnetic field was:

$$-\frac{\partial W}{\partial \Phi} = -\frac{1}{8\pi} \cdot EISH^2 \sin 2\Phi,$$

in which  $W$  is the energy of the disk in the magnetic field;  $\Phi$  the angle the normal to the disk makes with the lines of force;  $S$ , the effective area of the disk,  $\pi(r_2^2 - r_1^2)$ ; while  $E$ ,  $I$ , and  $H$  are as defined above.

The disk tends to turn in an opposite direction when the radial current is reversed, but not when the field is reversed.

For the radial current induced in the disk by the magnetic field, when no primary radial current is flowing, Corbino obtained the following expression,

$$Q = \frac{1}{4\pi\rho} SE_H H^2,$$

in which  $Q$  is the total quantity of electricity flowing radially in the disk,  $S$  and  $H$  as before,  $E_H$  the final value of the differential moment of the ions due to the field  $H$ , and  $\rho$  the resistance of the disk.

## 2. The Corbino Effect a True Hall Effect

Corbino considered the effect discovered by him to be distinct from the Hall effect. He claimed that, while the

Hall effect is due to the rotation of the equipotential lines in the magnetic field, in the circular disk carrying a radial current the equipotential lines are concentric circles, and that these lines are not bent or rotated.

Adams<sup>(6,7)</sup>, 1914, however, showed that the Corbino effect is, in reality, a Hall effect. He supposed a circular disk, Fig. 76, to have a hole in the centre, that a small sector is cut from the disk, and that the primary current enters by heavy wires,  $W, W$ , soldered to the edges of the disk. Since the resistance of the wires is low, the current density in the disk is inversely as the radius, and there is no radial current. The radii are the equipotential lines. If wires  $w, w$ , are

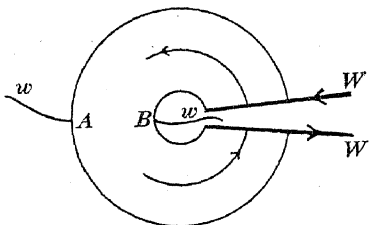


Fig. 76.

connected to a galvanometer, and the disk is placed in a magnetic field, a difference of potential will be set up at  $A$  and  $B$ . That is, a true Hall effect will be produced. Now the ordinary rectangular Hall plate may be con-

ceived of as bent into the Corbino circular disk, and when the primary current enters by one of the wires,  $W, W$ , a difference of potential is set up by the field at  $A$  and  $B$ . If the primary current enters and leaves by the wires,  $w, w$ , as in the Corbino case, the field will set up a difference of potential at the edges of the sector, and a circular current will result, when the sector is closed in. Hence the Corbino effect is in reality a Hall effect, due to a difference of potential set up in the disk by the magnetic field.

Adams developed expressions for the three Corbino effects in terms of the motions of negative electrons. He assumed that the electric current was carried entirely by free negative electrons, that these electrons in the absence of an electric field travel with equal velocities along zigzag paths, from collision to collision with the atoms of the metal, that

the collisions between electrons and atoms are similar to collisions between hard elastic spheres, that the external electric field causes the electrons to drift in a direction opposite to the field, and that the magnetic field causes the paths of the electrons to bend, so that there is a component at right angles both to the electric and to the magnetic field.

The expression for the circular current produced in the disk by the magnetic field he found to be,

$$C = -\frac{HeTI}{4\pi m} \log \frac{r_2}{r_1} = -\frac{e}{2\pi} HI \log \frac{r_2}{r_1},$$

in which  $e$  is Corbino's constant  $E$  (p. 126), and is equal to  $\frac{1}{2} \frac{e}{m} T$ ;  $H$ ,  $I$ ,  $r_2$ ,  $r_1$  are defined on p. 127,  $T$  is the free time of an electron, and  $e$  and  $m$  the charge and mass of an electron.

The expression for the torque according to Adams is:

$$-\frac{\partial W}{\partial \Phi} = -\frac{eH^2ITS}{16\pi m} \sin 2\Phi.$$

All the terms of the above expression have been already defined, p. 127.

For the radial current produced by the magnetic field, the expression is,

$$Q = -\frac{eS}{16\pi mR} \cdot H^2T,$$

in which  $R$  is the resistance of the disk, and the other terms as before.

Adams suggested that the Corbino effect is the fundamental galvanomagnetic effect, since it is independent of the thickness of the plate and of the free boundaries that introduce difficulties in the interpretation of the Hall effect. When a transverse current is led off from the edges of a rectangular Hall plate, the distribution of the current in the plate becomes complicated, and it is difficult to apply the electron theory to metallic conduction under these circumstances.



## 3. The Corbino Effect in Various Metals

The Corbino effect in a number of metals was measured by Adams and Chapman <sup>(8)</sup>, 1914. Their method was a modification of that of Corbino. In the following table are given the Corbino effects compared with that of copper as unity, and the relative Hall effects for the same metals, taken from the *Recueil de Constantes Physiques*, 1913. The Corbino effects are in terms of  $\frac{m}{H}$ , where  $H$  is the field strength, and  $m$  is computed from the equation,  $m = \frac{MC}{I}$ , in which  $M$  is the mutual inductance between the circular current  $C$  and the coil near the disk, and  $I$  is the primary radial current.

Metal	Hall Effect	Corbino Effect $\frac{m}{H}$	Metal	Hall Effect	Corbino Effect $\frac{m}{H}$
Cu . . .	-1'0	-1'0	Ag . . .	-1'60	-1'8
Fe . . .	+22'0	+2'65	Ni . . .	-9'04	-5'3
Al . . .	-0'73	-0'29	Zn . . .	+0'63	+0'13
Bi . . . ( $H=7200$ )	-10,700	-150'0	Cd . . .	+1'06	+0'27
Sb . . .	+421'0	+14'8	Co . . .	+4'6	+0'08
Pt . . .	-0'46	-0'075	Sn . . .	-0'077	<0'01
Au . . .	-1'27	-0'77	Pb . . .	+0'17	<0'01

The two effects have the same sign throughout, but the agreement in the order of magnitude is not always good.

Later Chapman <sup>(114)</sup>, 1916, pointed out that, since the Hall effect is computed as a difference of potential and the Corbino effect is measured as a current, if the values of the Corbino effects,  $\frac{m}{H}$ , referred to copper, are multiplied by the ratio of the specific resistances of the metals to copper,  $\frac{\rho}{\rho_{\text{Cu}}}$ , a better agreement is secured, as shown in the following table.

*Hall and Corbino Effects*

Metal	$H$	Hall Effect	$\frac{m}{H} \cdot \frac{\rho}{\rho_{Cu}}$
Cu . . . . .	Ind. $H$	— 1.0	— 1.0
Fe . . . . .	3700	+ 21.9	+ 16.4
Al . . . . .	Ind. $H$	— 0.73	— 0.48
Bi . . . . .	1650	— 19750	— 20100
" . . . . .	3930	— 12200	— 16300
Sb . . . . .	1750	+ 421	+ 557
Pt . . . . .	Ind. $H$	— 0.46	— 0.56
Au . . . . .	"	— 1.27	— 2.34
Ag . . . . .	"	— 1.60	— 1.61
Ni . . . . .	1700	— 23.7	— 32.0
Zn . . . . .	—	+ 0.63	+ 0.53
Cd . . . . .	Ind. $H$	+ 1.06	+ 1.37
Co . . . . .	3460	+ 4.61	+ 5.86

While the general agreement in the order of magnitude in the above results is fairly good, the lack of better agreement may be due to the fact that the two effects were determined in different specimens of the metals, and also may be due in part to the influence of the free boundaries on the Hall effect. In the case of the Hall effect, the presence of the Ettingshausen effect gives rise to a temperature difference at the edges of the plate, and this in turn, because of the Thomson effect, produces a difference of potential. While in the Corbino effect, the presence of the junction of different metals at the centre and at the circumference of the disk gives rise to a Peltier effect, and the Corbino effect becomes complicated by the radial heat flow, that in turn sets up a circular current in the presence of the magnetic field.

The following table gives the variation of the Corbino effect with the field strength for several metals as determined by Adams and Chapman <sup>(8)</sup>. The Corbino coefficient,  $c$ , is computed from the equation

$$C = \frac{c}{2\pi} \cdot \log \frac{r_2}{r_1} \cdot H I.$$

*Corbino Effects*

Metal	$H$	$c \times 10^7$	Metal	$H$	$c \times 10^7$
Copper . . .	7310	-2.38	Bismuth . .	30	-7.11
	6750	2.51	Antimony .	7210	+37.5
	5920	2.61		6180	38.3
	4220	2.40		4380	40.7
	2820	3.36		2590	40.9
Iron . . . . .	8450	+7.13	Platinum . .	7056	-0.199
	7620	6.60	Gold . . . .	6640	-1.99
	6750	6.58		5250	2.06
	4890	6.64	Silver . . . .	6000	-4.78
	3300	6.55		6240	5.05
	2060	8.27		5510	4.86
	1390	7.45		4580	4.84
Aluminium .	7970	-0.770	Nickel . . .	6930	-11.76
	6850	0.69		5970	13.0
	5660	0.83		4810	14.6
Bismuth . .	7720	-3.81		3730	14.5
	6540	4.15		2570	14.4
	5820	4.43	Zinc . . . . .	7470	+0.343
	4690	5.20	Cadmium .	7210	+0.715
	2630	6.13	Cobalt . . .	7570	+2.48
	953	6.66		6690	2.61
	419	7.71		5360	2.63
	107	7.39			

According to the above results, the Corbino effect,  $c$ , is nearly independent of the magnetic field, except in the case of bismuth, where the effect increases as the field decreases. However, in all the above metals the effect tends to decrease in higher fields.

The following table gives the results of Chapman's<sup>(114)</sup> determination of the Corbino effect and the Hall effect in a circular disc of *nickel*. The four electrodes for measuring the Hall effect were placed 90° apart.

Each effect at first increases with the field, reaches a maximum, and then decreases. The Corbino effect was found to be independent of the thickness of the disk.

*Corbino and Hall Effects in Nickel*

$H$	$e \times 10^7$	$H$	$R^a$
915	— 7'60	1100	— 0'01491
1830	10'60	2305	0'01448
2810	10'79	3420	0'01526
4580	10'07	5700	0'01581
6550	10'62	7750	0'01305
7050	9'45	9150	0'01127
9025	8'44	10200	0'00998

The Corbino effect in *iron* was found to rise rapidly with field strength at first. The effect reached a sharp maximum in a field of about 950 gauss, then diminished rapidly until a field of about 2000 gauss was reached, when the effect became linear and gradually decreased.

In order to find out whether the peculiar variation of the effect in iron could be attributed to a lack of uniformity in the magnetic field because of the 1 cm. hole in one of the pole pieces of the electromagnet, Chapman placed the iron disk and adjacent coils in a long solenoid, so that a uniform field might be secured.

With this new arrangement Chapman thought he found a reversal of the Corbino effect in a field of about 250 gauss. In a personal note to the writer, Prof. E. P. Adams, April 28, 1920, said: "I have found that Chapman's results on the Corbino effect in iron, the reversal in sign at low field, were wrong. Apparently the true effect was masked by a false effect arising from inductance in another part of the circuit. So it appears that there are no outstanding differences between the Hall effect and the Corbino effect that cannot be accounted for by experimental errors."

In *iron, nickel, cobalt, bismuth* and *antimony*, the Corbino and Hall effects are not proportional to the magnetic field strength. This may be accounted for by the fact that the free time of an electron is not constant, but depends on the strength of the external field.

According to Adams <sup>(7)</sup>, the Corbino effect, which implies an increase in resistance in the magnetic field, is related to this increase by the equation,

$$\frac{\Delta r}{r} = c^2 H^2,$$

in which  $r$  is the specific resistance of the metal, and  $c$  is the Corbino coefficient. This equation requires an increase in resistance in the magnetic field. Some metals, however, show a decrease. The Corbino effect cannot therefore be the sole cause of change of resistance in a magnetic field.

Smith (K. K.) <sup>(498)</sup>, 1916, determined the Corbino effect in several metals by measuring the torque produced by the magnetic field on disks of the metals carrying a radial current. The disks were suspended by a fine phosphor bronze wire, whose torsional coefficient was known. Readings were taken when the normal to the plane of the disk made an angle of  $45^\circ$  with the lines of force. The Corbino effect,

$E = \frac{1}{2} \frac{eT}{m}$ , was computed from the equation

$$-\frac{\partial W}{\partial \Phi} = -\frac{1}{8\pi} \cdot EISH^2 \sin 2\Phi.$$

The following table gives the results of Smith's determinations of  $E$  and  $T$ , and also those of Adams and Chapman.

Metal	$R$	$\rho$	$\frac{R}{\rho}$	$E$	$T$ (in Sec.)	$E$	$T$ (in Sec.)
				(Smith. K. K.)		(Adams and Chapman)	
Sb . . .	+0.192	$3.7 \times 10^4$	$+5.2 \times 10^{-6}$	$+3.91 \times 10^{-6}$	$4.42 \times 10^{-13}$	$+3.6 \times 10^{-6}$	—
Te . . .	+530	$1.5 \times 10^6$	$+3.5$ „	+1.48 „	1.66 „	—	—
Graphite	-0.695	$4.5 \times 10^5$	-1.5 „	$-5.74 \times 10^{-7}$	$6.5 \times 10^{-14}$	—	—
Ag . . .	-0.00088	$1.5 \times 10^3$	$-5.9 \times 10^{-7}$	-2.06 „	2.3 „	$-4.4 \times 10^{-7}$	$5.0 \times 10^{-14}$
W . . .	+0.00118	$6.2 \times 10^3$	+1.9 „	+1.56 „	1.8 „	—	—
Bi . . .	-10.1	$1.1 \times 10^5$	$-9.2 \times 10^{-5}$	$-7.4 \times 10^{-5}$	$8.40 \times 10^{-12}$	$-7.22 \times 10^{-5}$	—

The Hall effects above are those of Ettingshausen and Nernst <sup>(74)</sup>, except that for tungsten, which is due to A.W.Smith <sup>(493)</sup>. While  $R$  and  $E$  vary widely,  $\frac{R}{\rho}$  and  $E$  are of the same order of magnitude.

Heaps <sup>(252)</sup>, 1918, deduced the following equation for the Corbino current:

$$C = \frac{HITe}{2\pi m} \cdot \log \frac{r_2}{r_1},$$

in which the Corbino constant is  $E = \frac{eT}{m}$ ; while according to Adams,  $E = \frac{1}{2} \frac{eT}{m}$ . This latter is due to the assumption that the average drift velocity of the electrons is  $\frac{1}{2} X \frac{e}{m} T$ , and not  $X \frac{eT}{m}$ , where  $X$  is the electric force.

To test the following relation,

$$E = \frac{e}{m} T = \frac{R}{\rho},$$

in which  $R$  is the Hall coefficient, and  $\rho$  the specific resistance, Heaps devised a means by which he could determine the Corbino and Hall effects and the specific resistance in the same specimen of metal. The Corbino constant was found to be equal to the Hall coefficient divided by the specific resistance.

The Corbino effect in *copper* and *zinc* seemed independent of the field. In fields from 5000 to 17,000 gauss, for copper, mean  $E = -2.73 \times 10^{-7}$ , mean  $R = -4.25 \times 10^{-4}$ ; for zinc, mean  $E = +1.17 \times 10^{-8}$ , mean  $R = +6.66 \times 10^{-4}$ . The following are the results for *bismuth* and *graphite*.

Metal	$H$	$E$	$R$	$\rho$	$T$ secs.	$n$
Bismuth	800	$-10.9 \times 10^{-5}$	-15.4	$14.1 \times 10^4$	$6.16 \times 10^{-12}$	$6.09 \times 10^{18}$
	2450	-6.5	-10.0	15.4	3.67	9.38
	6150	-4.42	-7.85	17.7	2.50	11.9
	9750	-3.12	-6.40	20.5	1.76	14.6
	11100	-2.92	-6.15	21.1	1.65	15.2
	12850	-2.63	-5.92	22.5	1.49	15.8
Graphite	2600	$-6.15 \times 10^{-6}$	-0.612	$9.95 \times 10^4$	$3.47 \times 10^{-13}$	$1.53 \times 10^{20}$
	4200	-6.05	-0.662	10.95	3.42	1.42
	6550	-5.52	-0.760	13.8	3.12	1.23
	8400	-4.90	-0.810	16.5	2.77	1.15
	9900	-4.60	-0.890	19.4	2.60	1.05
	11300	-4.32	-0.946	21.8	2.44	0.99

In *bismuth*  $E$ ,  $R$ , and  $T$  decrease in higher fields, and  $\rho$  and  $n$  increase; while in *graphite*  $E$ ,  $T$ , and  $n$  decrease in higher fields, and  $R$  and  $\rho$  increase.

Puccianti<sup>(423)</sup>, 1918, by means of his specially devised double induction balance measured the Corbino effect in *bismuth* in weak fields. In fields from one hundred up to several hundred gauss, the Corbino effect increased a little more rapidly than the intensity of the field. At higher fields the effect increased less rapidly, and in fields above 1000 gauss, the effect decreased.

Trabacchi<sup>(547)</sup>, 1919, studied the relation between the Corbino and Hall effects in *bismuth*, and found that, at  $+20^{\circ}$  C. and  $-70^{\circ}$  C.,  $\frac{E}{Rc} = \text{constant}$ , in which  $c$  is the conductivity of the metal,  $E$  the Corbino coefficient, and  $R$  the Hall coefficient.

The variation of the Corbino effect with temperature in *bismuth* was measured by Collodi<sup>(125)</sup>, 1920. In moderate fields the effect was found to decrease rapidly with rise in temperature.

The effect fell to zero at the melting point of bismuth, and remained at zero in the molten metal. As the temperature fell below that of the room, the effect increased to twice that at room temperature. At  $-80^{\circ}$  C. the effect began to slowly decrease, and at  $-190^{\circ}$  C. had reached one half the room-temperature value.

#### 4. A Quasi-Corbino Effect in Electrolytes

Urbasch<sup>(552)</sup>, 1900, observed that when one electrolyte was placed above another in a flat dish, between a flat and a pointed pole of an electromagnetic, the liquid rotated as shown by the presence of lycopodium powder. The rotation continued as long as the field was on and as long as the two liquids had a separating surface. The rotation changed direction on the reversal of the field. The direction of ro-

tation also depended on the nature of the electrolytes used. When dilute  $HCl$  over  $CuCl_2$  was replaced by concentrated  $HCl$ , the rotation was reversed. For certain concentrations there was no rotation.

This rotation of electrolytes in a magnetic field seems due to the wandering of the ions across the boundary between the electrolytes. The ions in motion generate a circular field, and this field acted on by the divergent field of the magnet produces a rotation in the electrolyte.

The effect bears a strong resemblance to the Corbino effect in metals, p. 125.

Urbasch also placed a *zinc plate*, paraffined on one side, in *copper sulphate* solution, in a divergent magnetic field, and found that the disk rotated with a velocity of 5 to 10 cm. per sec. at the circumference.

He also investigated the influence of the magnetic field on chemical reactions, such as the dissolving of metals in acids, deposition of metals from salt solutions on the introduction of other metals. In every case the rotation of the electrolyte was perfectly regular and changed direction with the field.

Drude <sup>(151)</sup>, 1902, suggested that the rotation in the case of metals dissolving in acids was due to the deflection of the ions in a direction depending on their charges, and that the ions having the greater friction would carry the insulator with it. Drude, however, failed to observe any rotation in electrolytes. Urbasch claimed this to be due to his not employing a sufficiently strong field. Urbasch <sup>(552)</sup>, 1903, reported that Nernst had confirmed the rotation of electrolytes in a magnetic field as observed by him.

### 5. A Supposed Axial Hall Effect

La Rosa and Sellerio <sup>(460)</sup>, 1920, claimed to have observed a Hall effect in the direction of the lines of force of the magnetic field, and in a direction normal to the primary electric current through the metal.



Experiments were first made with a disk of *bismuth* 1.32 cm. thick and 2.7 cm. in diameter. The primary electrodes were fastened to diametrically opposite points on the edges of the disk. The secondary electrodes were point ones, fastened at the opposite centres of the circular surfaces of the disk. The plane of the disk was normal to the lines of force of the electromagnet. A current of 10 amperes was sent through the disk, and when the magnetic field was on, a difference of potential was observed at the secondary electrodes. This effect was found to reverse when either the primary current or the magnetic field was reversed. There existed, however, a decided lack of symmetry in the effect on the reversal of the field. The same axial effect was observed in *tellurium*.

Sellerio <sup>(476)</sup>, 1920, studied this axial effect in a rectangular plate of *bismuth*, with the electrodes placed in the usual Hall manner. The plate was so mounted that it could be rotated about the direction of the primary current as an axis. When the plane of the plate was normal to the lines of force, the usual Hall effect was observed. When, however, the plane made any angle other than  $90^\circ$  with the lines of force, the effect observed was supposed to contain a component of the true Hall effect and one of the axial effect.

Sellerio endeavored to explain mathematically that the Hall effect as usually measured consists of three components: 1. The true transverse Hall effect. 2. The axial effect. 3. The change in resistance.

The question naturally arises whether the axial effect may not be due to the fact that the magnetic lines of force were not all parallel to the line joining the secondary electrodes of the thick disk or plate, when in the axial position between the poles of the electromagnet, and hence the so-called axial effect is simply a residual part of the true Hall effect.

The asymmetry of the effect is probably simply that

observed in the Hall effect, and due to an asymmetrical change in resistance of the metal in the magnetic field.

Before the existence of this axial effect can be accepted as a distinct phenomenon, further investigation seems necessary.

## 6. The Hall and Kerr Effects

The Kerr <sup>(285)</sup> (1877) effect consists in a rotation of the plane of polarization of light when reflected from a magnetized surface. The rotation takes place both when the light is planepolarized in the plane of incidence and when perpendicular to this plane.

Shortly after the discovery of the Hall effect, Rowland <sup>(463)</sup>, 1880, proposed a theory of the Hall effect based on the electromagnetic theory of light. In this theory he considered the Hall and Kerr effects as fundamentally the same. Following the general method used by Maxwell in his electromagnetic theory of light, Rowland developed the following equations which include the electromagnetic action discovered by Hall:

$$A'' = a a \sqrt{b'^2 + c'^2} + b a' \sqrt{b^2 + c^2} + c (b c' - b' c) + d a a',$$

$$B'' = a b \sqrt{a'^2 + c'^2} + b b' \sqrt{a^2 + c^2} + c (c a' - c' a) + d b b',$$

$$C'' = a c \sqrt{a'^2 + b'^2} + b c' \sqrt{a^2 + b^2} + c (a b' - a' b) + d c c'.$$

In these equations,  $A''$ ,  $B''$ ,  $C''$  are the new E. M. F.s set up along the three axes by the magnetic field;  $a$ ,  $b$ ,  $c$  are the components of the force;  $a'$ ,  $b'$ ,  $c'$  the components of the electric current. Hall's discovery proved the existence of the constant  $c$ , and showed that  $a$  was very small or zero. The fact that the resistance of a metal is changed in a magnetic field confirms the existence of  $b$  and  $d$ .

Rowland concluded his article in these words: "Thus the new electromagnetic phenomenon [Hall effect] explains in the most perfect manner the magnetic rotation of the plane of polarization of light, and we are almost in the position to pronounce positively that the two phenomena are the same. Should this preliminary theory of the subject stand the test

of time, it hardly seems to me that we can regard it in any other light than a demonstration of Maxwell's theory of light."

Rowland's theory requires that all magnetic substances should have the same sign for the Hall effect, and that they should rotate the plane of polarization in the same direction. However, the sign of the Hall effect in *iron* and *cobalt* is positive, while in *nickel* the effect is negative. Hall <sup>(237)</sup>, 1881, found that *nickel* and *cobalt* rotated the plane of polarization in the same direction as *iron*, while magnetite has been found to rotate the plane in the opposite direction.

Van Schaik <sup>(470)</sup>, 1882, in a note appended to his article on the electromagnetic rotation of the plane of polarization, agrees with Rowland that the Hall phenomenon and the magneto-optic rotation are due to the same cause.

In 1884, Lorentz <sup>(338)</sup> developed the following expression for the Hall current through a galvanometer connected with the secondary electrodes of a metal plate:

$$i = \frac{hE}{d \left( R + \frac{a}{d} \right) \left( r_g + \frac{a'}{d} \right)} \dots \dots \dots (1)$$

where  $h$  is a factor depending on the magnetic field and the nature of the metal,  $E$  is the primary electromotive force,  $R$  and  $r_g$  the resistances respectively of the external circuit and the galvanometer circuit,  $\frac{a}{d}$  and  $\frac{a'}{d}$  the respective resistances of the plate opposed to the primary and secondary currents, and  $d$  the thickness of the plate.

According to Lorentz the absolute value of the Kerr effect can be deduced from the Hall effect by means of the following expression:

$$C_p = \frac{h'}{2T} \cdot \frac{\sigma^3}{1 + 2\sigma \cos \tau + \sigma^2} \dots \dots \dots (2)$$

where  $C_p$  is the amplitude of the component in question of the reflected light,  $h'$  the constant including the Hall coefficient, to be computed from the Hall equation (1) above, and cor-

rected for what he terms the dielectric polarization capacity of the air,  $T$  the period of vibration, and  $\sigma$  and  $\tau$  optical constants of the reflecting surface.

Goldhammer<sup>(215)</sup>, 1892, and Drude<sup>(148)</sup>, 1892, each developed sets of equations to represent the phenomena of magneto-optic reflection. Their equations include terms representing the Hall effect.

According to Sir J. J. Thomson<sup>(518)</sup>, 1893, the effect established by Hall for steady currents is of the same nature as that found by Kerr for rapidly alternating currents, which constitute light.

"Here, however, the resemblance ends; the values of the coefficient  $C [= R]$  deduced by Hall from his experiments on steady currents do not apply to rapidly alternating light currents. Thus Hall found that for steady currents the sign of  $C$  was positive for iron, negative for nickel; the magneto-optical properties of these bodies are, however, quite similar. Again Hall and Righi found that  $C$  for bismuth was enormously larger than that for iron or nickel. Righi, however, was unable to find any traces of magneto-optical effects in bismuth. The optical experiments previously described show that there is an electromotive intensity; they do not, however, show without further investigation on what function of the electromotive intensity the magnitude of the transverse intensity depends. Thus, for example, the complete current in the metal is the sum of the polarization and conduction currents. Thus, if the electromotive intensity is  $X$ , the total current  $u$  is given by the equation

$$u = \left( \frac{K'}{4\pi} \cdot \frac{d}{dt} + \frac{1}{\sigma} \right) X,$$

or if the effects are periodic and proportional to  $e^{ipt}$ ,

$$u = \left( \frac{K'}{4\pi} \cdot ip + \frac{1}{\sigma} \right) X,$$

where  $K'$  is the specific inductive capacity of the metal and  $\sigma$  its specific resistance.

We do not know from the experiments, without further discussion, whether the transverse electromotive intensity is proportional to  $u$ , the total current, or only to  $\frac{K'ipX}{4\pi}$ , the conduction current." — — — — —

"Hence of the three hypotheses, (1) that the transverse electromotive intensity concerned in these magnetic optical effects is proportional to the total current, (2) that it is proportional to the polarization current, (3) that it is proportional to the conduction current, we see that (1) and (3) are inconsistent with Kerr's experiments on the reflection from tangentially magnetized mirrors, while (2) is completely in accordance with them.

"The transverse electromotive intensity indicated by hypothesis (2) is of a totally different character from that discovered by Hall. In Hall's experiments the electromotive intensities, and therefore the currents through the metallic plates were constant; when, however, this is the case the 'polarization' current vanishes. Thus in Hall's experiments there could have been no electromotive intensity of the kind assumed in hypothesis (2); there is therefore no reason to expect that the order of the metals with respect to Kerr's effect should be the same as that with respect to Hall's."

Starting with Maxwell's equations for the electromagnetic theory of light, Wind <sup>(599)</sup>, 1894, developed a mathematical theory to account for the connection between the Hall effect, the Kerr effect, and the Faraday effect.

Hopkinson <sup>(267)</sup>, Lorentz <sup>(338)</sup>, and others had assumed that the Hall effect could be represented by the relation,

$$F = \frac{1}{p} C - q [HC],$$

in which  $F$  is the electric force,  $C$  the current,  $H$  the magnetic force,  $p$  a complex constant depending on the medium and the period  $T$ , and  $q$  a real constant dependent upon the

properties of the medium. Wind (l. c.) suggested that, in order to explain completely the magneto-optical phenomena,  $q$  must be a *complex* constant.

For the phase differences of the magneto-optical phenomena, he found for the polar case:

$$m_p = m_i - \pi = 4\tau - \delta_1 - \delta_2 - \delta_4 - \frac{1}{2}\pi + S;$$

and for the equatorial case:

$$m_p = m_i = 3\tau - w - \delta_1 - \delta_2 - \delta_4 - \frac{1}{2}\pi + S.$$

In the above equations  $S$  is called the argument of the Hall effect, and is assumed to be complex. The Hall constant is  $qe^{iS}$  or  $qN$ . The total current  $C$  is supposed to be composed of two parts, one a conductive current  $C_1$ , and the other a displacement current  $C_2$ . Each is supposed to give rise to a Hall effect, respectively  $h$  and  $k$ , given by the following equations:

$$h = \frac{q}{N} \cos S + \frac{2\pi}{NT} \cdot \frac{p_2}{p_1} q \sin S,$$

$$k = \frac{q}{N} \cos S - \frac{T}{2\pi N} \cdot \frac{p_1}{p_2} q \sin S.$$

According to Lorentz (l. c.)  $h$  and  $k$  are equal. This equality follows, if  $S$  is assumed to be zero. According to Drude<sup>(148)</sup> and Thomson<sup>(518)</sup>  $h$  is zero, that is, the conductive current in optical phenomena produces no Hall effect.

Leathem<sup>(512)</sup>, 1897, developed equations for the magneto-optic phenomena somewhat similar to those of Drude and others. His equations (10 and 11) contain Hall effect terms for conduction and displacement currents. If  $C$  is the Hall coefficient, then the equations are of the type:

$$P = P' + C(\beta'w - \gamma'v),$$

where  $P, Q, R$  are the components of the electromotive force, of the magnetic force, and  $u, v, w$  of the total current. If  $b_1, b_2, b_3$  become zero, then

$$C\beta' = -\frac{4c^2\lambda^2}{R^4e^{4\mu}} \cdot g_2.$$

If the value of  $C$  for iron and the values of  $R, \alpha, \lambda$  for sodium light are substituted in the above equation,  $g_2$  will have much too small a value to account for the Kerr effect.

However, Leatham saw no reason why the Hall effect for rapidly alternating currents, such as those of light, should not be very large compared with that for steady currents. "There is nothing unnatural in this, for the incipient conductions which make optical opacity have no relation of continuity whatever with the steady conduction in an ordinary current; thus Maxwell found that the ordinary coefficients of 'conductivity' are very much smaller in the optical circumstances. And it may be noticed that, as  $C$  is proportional to the electromotive force divided by the current, a greatly diminished conductivity will correspond to a greatly increased value of  $C$ ."

Larmor<sup>(302)</sup>, 1900, deduced for the magneto-optic phenomena equations for the components of the total current  $u, v, w$ , of the type:

$$u = (4\pi c^2)^{-1} \left( k' \frac{dP}{dt} - \alpha_3 \frac{d^2 Q}{dt^2} + \alpha^2 \frac{d^2 R}{dt^2} \right) - \alpha_3 Q + \alpha_2 R,$$

in which  $P, Q, R$ , are components of the electric force, and  $\alpha_1, \alpha_2, \alpha_3$ , of the Hall effect.

## 7. The Hall and Faraday Effects

About 1845 Faraday<sup>(294)</sup> discovered that isotropic substances of high refractive index, when placed in a magnetic field, rotate the plane of polarization of plane polarized light. The amount of rotation,  $\Theta$ , is proportional to the field strength,  $H$ , to the length of the path traversed,  $l$ , and is given by the relation:

$$\Theta = rHl,$$

where  $r$  is known as Verdet's constant.

Élie<sup>(263)</sup>, 1882, suggested that the Hall rotation of equipotential lines and the Faraday rotation of the plane of polarization by a magnetic field were similar phenomena, and

may be compared with the rotation of a gyroscope. The rotation in each case is the resultant of two forces. The Hall constant may be said to be proportional to the moment of inertia of the material molecule.

Kundt <sup>(302)</sup>, 1884, 1886, found that thin transparent sheets of *iron*, *cobalt* and *nickel*, when placed normal to a magnetic field, rotated very strongly the plane of polarization of light, and this rotation was proportional to the magnetization and not to the strength of the magnetic field.

Du Bois <sup>(77)</sup>, 1887, observed similar results in *cobalt* and *nickel*. He pointed out the fact that Maxwell's equations for the electromagnetic theory of light and Rowland's <sup>(463)</sup> equations based on the relation between magneto-optic phenomena and the Hall effect were in accord with the Faraday-Verdet experimental law.

Sir J. J. Thomson <sup>(518)</sup>, 1893, in discussing the work of Kundt, said: "The effect of a magnetic field in producing rotation of the plane of polarization thus seems to afford strong evidence of the existence of a transverse electromotive intensity in a conductor placed in a magnetic field, this intensity being quite distinct from that discovered by Hall, inasmuch as the former is proportional to the rate of variation of the electromotive intensity, whereas the Hall effect is proportional to the electromotive intensity itself."

Using the magnitude of this transverse intensity, for the conditions under which Kundt worked, Thomson found that the ratio of the magnitude of the transverse intensity to that producing the current to be for iron

$$1.6 \times 10^{-17} p.$$

The factor  $p$  is so small that the transverse force will be insensible unless the electromotive intensity is changing with a rapidity comparable with that occurring in ordinary light. Hence this transverse electromotive intensity produces a measurable effect only in optical phenomena.

An equation giving the relation between the Hall and



Faraday effects was developed by Wind <sup>(599, 600)</sup>, 1894—1898. Verdet's constant,  $r$ , is given by the equation,

$$r = \frac{n^3}{\lambda^2} \cdot \frac{q}{v} 5400,$$

where  $n$  is the index of refraction of the medium,  $\lambda$  the wavelength and  $v$  the velocity of light, and  $q$  is the Hall constant per unit magnetic force.

For *carbon disulphide*, in which  $r$ ,  $n$ ,  $\lambda$ ,  $v$  are known, the Hall effect is,  $q = 1.92 \times 10^{-4}$ , which is of the same order of magnitude as the Hall effect in some metals.

Drude <sup>(148, 153)</sup>, 1892, 1900, developed equations for the rotation of the plane of polarization of transmitted light in a magnetic field. Those equations were based on the conception of movable ions and contained Hall effect terms.

## CHAPTER IX

### THE ETTINGSHAUSEN EFFECT: TRANSVERSE GALVANOMAGNETIC TEMPERATURE DIFFERENCE

#### 1. Historical and General

The similarity which Boltzmann <sup>(86)</sup>, 1886, found between the properties of gases and of bismuth in a magnetic field, led him to the supposition that the shifting of the cathode rays in a Geissler tube in a magnetic field should find its analogue in the displacement of heat in a bismuth plate through which an electric current was flowing.

Ettingshausen <sup>(169)</sup>, 1887, found by actual experiment that a plate of *bismuth* carrying an electric current, when placed in a magnetic field at right angles to the lines of force, suffered an unequal change in temperature at its lateral edges. He used a plate of bismuth 3.1 cm. long, 2.4 cm. wide, and about 0.04 cm. thick. To the ends of the plate were soldered heavy copper wires,  $C$ ,  $C$ , Fig. 77. At the middle of the

lateral edges were soldered two thermo-couples,  $t, t$ , for detecting changes in temperature. When the electric current flowed through the plate and the magnetic field was on as indicated in the figure, the edge  $A$  became cooler, and  $B$  warmer.

This change in temperature was reversed both on the reversal of the primary current and of the magnetic field.

The transverse change in temperature per unit width  $\frac{\Delta T}{b}$  was found to be proportional to the magnetic field  $H$ , the current density  $i$  of the primary electric current  $I$ , and the width of the plate  $b$ .

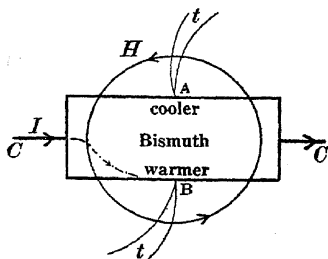


Fig. 77.

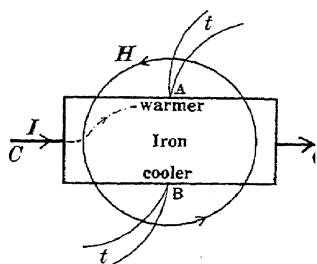


Fig. 78.

$$\text{Hence } \frac{\Delta T}{b} = PHi. \text{ But } i = \frac{I}{bd}.$$

Therefore  $\Delta T = \frac{PHI}{d}$ , in which  $P$  is the Ettingshausen coefficient, and  $d$  the thickness of the plate.

#### Sign of the Ettingshausen Effect

The effect in *bismuth* is said to be *positive*, according to this convention: If in going from the point of entrance the current  $I$ , Fig. 77, to the warmer point  $B$ , the passage  $IB$  is in the direction of the Amperian current of the field  $H$ , the effect is termed *positive*. If the passage to the warmer point  $A$ , Fig. 78, is opposed to the Amperian current, the effect is said to be *negative*. This is the case in *iron*. Ettingshausen in his first investigations determined the effect

*bismuth*, *tellurium*, *antimony*, *iron*, *nickel* and *cobalt*. The effect was found to be positive in all these metals, except in iron.

Boltzmann <sup>(87)</sup>, 1887, deduced the following expression for the Ettingshausen difference of temperature:

$$\Delta T = \frac{aHI}{Kd},$$

in which  $a$  is a constant depending on the metal, and  $K$  the thermal conductivity.

Comparing this equation with

$$\Delta T = \frac{PHI}{d},$$

we have  $P = \frac{a}{K}$ , and according to this, the Ettingshausen effect varies inversely with the thermal conductivity of the metal.

In 1887, Ettingshausen and Nernst <sup>(173)</sup> determined the Ettingshausen change in temperature in *bismuth* and *bismuth-tin alloys*, with the following results:

Plate	Bismuth	Tin	Thickness	$H$	$I$	$\Delta T$
Bi . . .	100%	0 %	0.093 cm.	9360	0.561 Abs.	3.24° C.
L <sub>1</sub> . . .	99.05	0.95	0.088	9310	0.655	2.46°
L <sub>2</sub> . . .	98.54	1.46	0.115	9390	0.622	0.961°
L <sub>3</sub> . . .	93.86	6.14	0.110	9330	0.659	0.442°
L <sub>4</sub> . . .	86.9	13.1	0.080	9350	0.685	0.073°

Thus it is seen that the Ettingshausen effect decreases rapidly as the tin content increases.

Van Everdingen <sup>(182)</sup>, 1898, found that the Ettingshausen difference of temperature set up by the magnetic field in *bismuth* increased rapidly at first, then more slowly, and reached its maximum value in about one minute.

## 2. Correction of the Ettingshausen Effect for Thermal Conductivity

Zahn <sup>(609)</sup>, 1904, pointed out that the Ettingshausen coefficient,  $P$ , must be corrected for the thermal conductivity of the surrounding material, in accordance with the following expression:

$$P = \frac{1}{2} \beta \frac{u-v}{e^{b\beta} - e^{-b\beta}} \cdot \frac{\Delta T \cdot d}{HI},$$

in which  $b = \sqrt{\frac{h}{2kd}}, u = (h + 2kb)e^{b\beta},$

and  $v = (h - 2kb)e^{-b\beta},$

where  $h$  is the heat constant given to the surroundings, and  $k$  the thermal conductivity of the plate. Applying this correction Zahn obtained the following results for two plates of *antimony*:

Plate	Temperature	$\beta$	$d$	$P$ uncorrected	$P$ corrected
Antimony I.	18° C.	2.40 cm.	0.0512 cm.	$17.78 \times 10^{-7}$	$19.63 \times 10^{-7}$
„ II.	„	1.38	0.0507	18.68	19.40

The corrected value is 9 to 10 per cent. larger than the uncorrected.

### 3. Correction of the Ettingshausen Effect for the Peltier and Righi-Leduc Effects

Gottstein <sup>(218)</sup>, 1914, has shown that the Ettingshausen effect should be corrected for the Peltier effect, and for the Righi-Leduc effect set up in consequence of the Peltier heat.

He deduced the following expression for the corrected coefficient  $P$ :

$$P_{corr.} = P - \frac{2 S II}{k},$$

where  $S$  is the Righi-Leduc coefficient, p. 235,  $II$  the Peltier heat, and  $k$  the thermal conductivity of the metal.

### 4. Variation of the Ettingshausen Effect with Temperature and Magnetic Field

Zahn <sup>(609)</sup>, in 1904, observed that the Ettingshausen effect in *bismuth* increased from  $3.53 \times 10^{-5}$  to  $6.27 \times 10^{-5}$ , when the magnetic field increased from 3930 to 10,570 gauss. In 1911, Hall and Campbell <sup>(247)</sup> found that the effect in soft iron increased from  $4.92 \times 10^{-8}$  to  $6.12 \times 10^{-8}$ , when the temperature rose from 12.2° to 83.9° C.

According to Senepa<sup>(479)</sup>, 1913, the effect in *bismuth* seemed to be independent of the field, Fig. 79, until  $H$  reached a value of 1000 gauss, and then the effect appeared to fall off gradually as the field further increased.

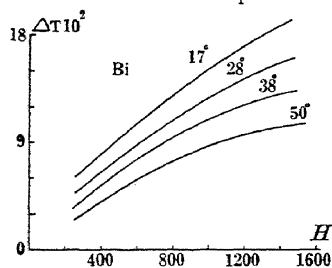


Fig. 79.

There was a decided decrease in the effect as the temperature rose from 17° to 50° C.

The following values for the Ettingshausen effect in poor conductors were determined by Königsberger and Gottstein<sup>(296)</sup>, 1913.

	Temperature	$P$	$\frac{P}{P'}$	$\frac{N}{N'}$
Graphite . . . . .	16° C.	$-5.9 \times 10^{-5}$	1.08	1.14
	90°	$-5.5$		
Silicon + 5% Fe	16°	$+2.6 \times 10^{-2}$	1.39	1.42
	90°	$+1.9$		
Molybdenite . . .	18°	$+2.8 \times 10^{-2}$	3	2.13
	90°	$+0.9$		

In each substance above, the effect diminished with increase in temperature. The ratio,  $\frac{P}{P'}$ , of the effects at the two temperatures, was nearly the same as the ratio,  $\frac{N}{N'}$ , of the electron numbers in a unit volume at these temperatures. Thus there would seem to be some intimate connection between the Ettingshausen effect and the electronic pressure in the material.

The Ettingshausen effect in *tellurium* was found by Wold<sup>(604)</sup>, 1916, to increase with the temperature, up to 100° C. Beyond this temperature, the increase was less rapid, as seen below.

*Ettingshausen Effect in Tellurium*

Temperature	$P$	Temperature	$P$
13° C.	$+7.19 \times 10^{-5}$	79° C.	$+25.6 \times 10^{-5}$
27°	10.45	124°	36.1

The effect was independent of the field.

Smith <sup>(493)</sup>, 1916, studied the Ettingshausen effect in the alloys, *Bi-Sn*, *Bi-Pb*, *Bi-Cd*, and *Bi-Sb*, and noticed that the addition of small or large quantities of lead or tin to bismuth produced a rapid decrease in the effect, while small additions of antimony or cadmium caused an increase in the effect. Larger amounts of the latter metals produced a decrease in the effect.

In the case of the *lead*, *tin*, and *cadmium alloys* the effect was not proportional to the field strength.

## . Theories of the Ettingshausen Effect

*a. Riecke's Theory*

Employing the same suppositions in regard to the motions of the positive and negative ions as in his theory for the Hall effect, p. 78, Riecke <sup>(438)</sup>, 1898, deduced the following expression for the difference of temperature set up by the magnetic field at the upper and lower edges of the plate:

$$\Delta T = k \cdot \frac{v_p v_n (v_p + v_n)}{v_p w_n + v_n w_p} \cdot \frac{Hbi}{\sigma}.$$

Hence the Ettingshausen coefficient is,

$$P = \frac{k}{\sigma} \cdot \frac{v_p v_n (v_p + v_n)}{v_p w_n + v_n w_p},$$

in which  $k$  is the thermal conductivity and  $\sigma$  the electrical.

In 1906, Riecke <sup>(440)</sup> developed a somewhat different expression for the Ettingshausen coefficient, as follows:

$$P = \frac{1}{6 \left( 1 + \frac{4}{3} T \frac{d \lg n_1 n_2}{dT} \right)} \cdot \frac{e^2}{c^2 \alpha^2} \cdot \frac{v_1 l_1 + v_2 l_2}{\sigma T},$$

in which  $v_1$  and  $v_2$  are the velocities,  $l_1$  and  $l_2$  the mean free

paths, and  $n_1$  and  $n_2$  the numbers per unit volume respectively of the positive and negative ions.

*b. Drude's Theory*

Drude <sup>(149)</sup>, 1900, starting with the assumption that an electric current is carried by two kinds of free electrons, positive and negative, deduced the following expression for the Ettingshausen effect:

$$P = \frac{1}{pe(v_1 N_1 + v_2 N_2)} \cdot \frac{(v_1 + v_2)}{\frac{dN_1}{dT} + \frac{dN_2}{dT}},$$

in which  $p$  is a universal constant, and the other letters have the same significance as in the Hall effect equation of Drude on p. 80. According to this dualistic theory, the Ettingshausen effect depends on the velocities of the two kinds of electrical particles, and on their volume density, which changes with temperature.

*c. Zahn's Theory*

Zahn <sup>(613)</sup>, 1913, as already stated under the Hall effect, p. 87, supposed both the vibratory and migratory velocity of the positive particle in the conductor to be zero, and on this supposition he developed the following expression for the Ettingshausen effect:

$$P = \frac{3}{4 \alpha T N_1 (\Phi_1 + \Phi_2)},$$

in which  $\Phi_1$ ,  $\Phi_2$ ,  $T$ ,  $N_1$ , have the same meaning as in the Hall effect equation, p. 87, and  $\alpha$  is a universal constant. In this case the effect  $P$  varies inversely as the temperature  $T$ , and the electronic density. As already noted, Senepa, p. 150, found that  $P$  decreased in *bismuth* with rise in temperature; while Wold, p. 150, found that in *tellurium* the effect increased with temperature.

*d. Livens' Theory*

On the supposition that the electric current in a conductor is carried by the negative electrons alone, Livens <sup>(329)</sup>,

1915, developed the following expression for the Ettingshausen effect:

$$\frac{d\Theta}{dz} = \frac{e^2 l_m}{mcR} \cdot H E_y \left\{ \frac{\left( \frac{2}{S} - \frac{1}{2} \right) \Gamma \left( \frac{3}{2} + \frac{4}{S} \right)}{2 + \frac{2}{S}} \Gamma \left( 2 + \frac{2}{S} \right) \right\},$$

in which  $e$ ,  $l_m$ ,  $m$ ,  $c$ ,  $H$  and  $S$  have the same meaning as in the Hall effect, p. 87.  $E_y$  is the E. M. F. of the primary current, and  $R$  is the usual absolute constant, in the equation

$$q = \frac{m}{2R\Theta}.$$

If the Hall E. M. F.,  $E_z$ , p. 87, is substituted in the above expression, we shall have

$$\frac{d\Theta}{dz} = \frac{S-4}{4(S+1)} \cdot \frac{e}{R} \cdot E_z.$$

This latter expression agrees with that of Bohr<sup>(75)</sup>, 1911, who pointed out that this expression changes sign for different values of  $S$ .

If  $S$  is  $< 4$ , the Ettingshausen and Hall effects have the same sign; while if  $S$  is  $> 4$ , the signs of the two effects are opposite in character, as is the case in bismuth. This relative value of  $S$  seems to furnish some evidence as to the force exerted between electrons and molecules.

On p. 90 Hall has proposed a possible explanation of the Ettingshausen effect and its change of sign.

## 6. Relation Between the Ettingshausen and Hall Effects

Ordinarily the Hall coefficient is computed from the equation

$$R = \frac{Ed}{HI},$$

in which  $E$  is the total difference of potential observed at the Hall electrodes on the edges of the plate. The Ettingshausen difference of temperature at the edges of the plate sets up a thermo-electromotive force which must be added to, or subtracted from, the observed  $E$ . So that the Hall



$E. M. F. = E_{obs.} \pm \Delta T \cdot \Theta$ , where  $\Delta T$  is the Ettingshausen difference of temperature, and  $\Theta$  is the thermoelectric power of the plate against the lead wires. If the lead wires are of the same metal as the plate, and are in the same physical state, the observed  $E$  needs no correction in computing the Hall effect. Such leads as the above are hard to obtain. So it is better to correct the  $E_{obs.}$  for the thermal E. M. F.

If  $E_R$  is the true Hall E. M. F., then the Hall equation,

$$R = \frac{E_R \times d}{HI},$$

becomes 
$$R = \frac{E_{obs.} \times d}{HI} \pm \frac{\Delta T \cdot d \cdot \Theta}{HI},$$

or 
$$R = \frac{E_{obs.} \times d}{HI} \pm P \times \Theta,$$

where  $P$  is the Ettingshausen coefficient.

Zahn <sup>(509)</sup>, 1904, starting with Drude's <sup>(149)</sup> equations for the Hall and Ettingshausen effects, obtained the following relation between the two effects:

$$\frac{R}{P} = 0.5 \times 10^{-2} \text{ p. c. } \frac{v_1 - v_2}{v_1 + v_2},$$

in which the universal constant  $p = 1.656 \times 10^{-4}$ ,  $c = 3 \times 10^{10}$ , and  $v_1$  and  $v_2$  are the respective velocities of the negative and positive electrons. If  $v_2$  with respect to  $v_1$  is neglected, then

$$\frac{R}{P} = 2.5 \times 10^4.$$

The following table contains results observed by Zahn.

Metal	$\frac{R}{P}$	Metal	$\frac{R}{P}$
Bismuth I . .	$7.7 \times 10^4$	Nickel I . . .	$16.7 \times 10^4$
„ II . .	3.4 „	„ II . . .	7.1 „
„ III . .	3.3 „	Iron . . . . .	18.8 „
Antimony . .	11.3 „	Cobalt . . . .	18.0 „

From the above it is seen that the ratio obtained from observed values of  $R$  and  $P$  is of the same order of magnitude as the ratio obtained from theoretical considerations.

Königsberger and Gottstein <sup>(297)</sup>, 1915, employing the equations of Gans <sup>(208)</sup>, showed that the following relation should hold between the Ettingshausen and the isothermal Hall coefficient:

$$P = \frac{3}{8} \frac{\alpha}{e} R_i = -2.9 \times 10^{-5} R_i.$$

The table below shows that  $P$  calculated from this equation does not agree with the observed values, either in magnitude or sign.

Metal	$P_{calc.}$	$P_{obs.}$
Iron . . . . .	$+3.1 \times 10^{-8}$	$-5.7 \times 10^{-8}$
Nickel . . . . .	$-1.3 \times 10^{-7}$	$+2.8 \times 10^{-8}$
Antimony . . . .	$+5.8 \times 10^{-6}$	$+1.3 \times 10^{-6}$

Starting with the equations of Gans <sup>(208)</sup>, Zahn <sup>(615)</sup>, 1915, developed the following relation between the Hall adiabatic and isothermal effects and the Ettingshausen effect:

$$R_a - R_i = -(1.721 \times 10^4 - \sigma) P,$$

where  $\sigma$  is the Thomson effect in ergs per electro-magnetic units. Since  $\sigma$  is small compared with  $1.721 \times 10^4$ , we can write

$$R_a - R_i = -1.721 \times 10^4 P.$$

The following relation between  $R_a$ ,  $R_i$ , and  $P$  was deduced by Heurlinger <sup>(259)</sup>, 1915:

$$R_a - R_i = -E_i H^2 P,$$

in which  $E_i$  is the longitudinal thermomagnetic difference of potential.

### 7. Relation Between the Ettingshausen and Righi-Leduc Effects

If the compensation method of Barlow <sup>(44)</sup> is employed, the electric and heat currents may be so sent through the plate that the Ettingshausen and the Righi-Leduc (p. 235) temperature changes at the edges of the plate annul each other. Then we shall have

$$0 = SHb \frac{dt}{dl} + \frac{PHI}{d},$$

whence

$$\frac{S}{P} = -I \div b d \frac{dt}{dl}.$$

Since the current density  $i = \frac{I}{bd}$ , then

$$\frac{S}{P} = -i \div \frac{dt}{dl}.$$

Barlow obtained the following values for  $\frac{S}{P}$ .

Metal	$H$	$\frac{dt}{dl}$	$i$	Temperature	$\frac{S}{P}$
Antimony .	9130	10° C.	0·82	44° C.	+ 0·0817
Bismuth . .	9030	12·3°	0·99	50°	— 0·081
„ . .	9030	18·5°	1·57	42°	— 0·085

### 8. Ettingshausen Effects

Substance	Temperature	Field	$P$	Observer
Aluminium . .	Room temp.	—	+ 1·06 × 10 <sup>-9</sup>	Unwin
Antimony . .	18° C.	Ind. $H$	+ 1·94 × 10 <sup>-6</sup>	Zahn
	20°	9130	+ 2 × 10 <sup>-6</sup>	Barlow
Bismuth . . .	15°	2800	+ 5·7 × 10 <sup>-5</sup>	Ettingshausen and Nernst
	15°	4710	+ 5·8 „	„
	14°	10000	+ 10 „	Barlow
„ I. . .	18°	3930	+ 3·53 × 10 <sup>-5</sup>	Zahn
	„	6290	+ 5·16 „	„
	„	8300	+ 5·83 „	„
	„	10570	+ 6·27 „	„
„ II. . .	„	2660	+ 3·67 „	„
	„	6290	+ 5·81 „	„
	„	8300	+ 6·20 „	„
	„	10670	+ 6·34 „	„
Bismuth . . .	44°	300 to 1200	+ 3·31 „	Senepa
	23°	3000 to 14000	+ 7·34 „	Smith, A. W.
	Room temp.	2730	$\Delta T + 0·03°$ C.	Everdingen
	„	4800	+ 0·16° C.	„
	„	6100	+ 0·29°	„
Brass . . . . .	—	—	Too small to observe	Zahn
Cadmium . .	Room temp.	—	— 2·87 × 10 <sup>-9</sup>	Unwin
Carbon . . . .	90° C.	10270	+ 0·5 × 10 <sup>-5</sup>	Zahn
Cobalt . . . .	24°	4400	+ 0·9 × 10 <sup>-8</sup>	„
	Room temp.	—	+ 2·16 „	Unwin

Substance	Temperature	Field	$P$	Observer
Constantan . .	—	—	Too small to measure	Zahn
Copper . . . .	Room temp.	—	$-1.60 \times 10^{-9}$	Unwin
	—	—	Too small to measure	Zahn
German Silver	—	—	"	"
Graphite . . .	$17^{\circ}$ C.	5460	$-5.98 \times 10^{-5}$	Gottstein
" . . .	$90.2^{\circ}$	"	$-5.46$ "	"
Heusler Alloy	—	—	$< 3 \times 10^{-8}$	Zahn and Schmidt
Iridium . . .	—	—	Too small to measure	Zahn
Iron . . . . .	$18^{\circ}$	6290	$-5.7 \times 10^{-8}$	"
	$12.2^{\circ}$	5000	$-4.92$ "	Hall and Campbell
	$83.9^{\circ}$	"	$-6.12$ "	"
	Room temp.	—	$-4.26$ "	Unwin
Steel . . .	$20^{\circ}$	—	$-6.7$ "	Zahn
Molybdenite .	$18.1^{\circ}$	5460	$+2.77 \times 10^{-2}$	Gottstein
	$91.3^{\circ}$	"	$+6.98 \times 10^{-3}$	"
Nickel I . . .	$18^{\circ}$	10620	$+2.8 \times 10^{-8}$	Zahn
" II . . .	"	6290	$+1.76 \times 10^{-7}$	"
	Room temp.	—	$+3.03 \times 10^{-8}$	Unwin
Palladium . .	—	—	Too small to measure	Zahn
Platinum . . .	—	—	"	"
Silicon . . . .	$18.7^{\circ}$	6150	$+2.65 \times 10^{-3}$	Gottstein
	$89.7^{\circ}$	"	$+1.72$ "	"
	$60^{\circ}$	—	$< 3 \times 10^{-6}$	Buckley
Silver . . . . .	—	—	Too small to measure	Zahn
	Room temp.	—	$-1.65 \times 10^{-9}$	Unwin
Tellurium . .	$65^{\circ}$	5500	$+1.4 \times 10^{-4}$	Lloyd
	"	"	$+2.9$ "	"
	—	—	$+2.0$ "	Ettingshausen
	$13^{\circ}$	1000 to 15000	$+7.19 \times 10^{-5}$	Wold
	$27^{\circ}$	"	$+10.45$ "	"
	$79^{\circ}$	"	$+25.6$ "	"
	$124^{\circ}$	"	$+36.1$ "	"
Zinc . . . . .	—	—	Too small to measure	Zahn
	Room temp.	—	$-2.67 \times 10^{-9}$	Unwin

## CHAPTER X

GALVANOMAGNETIC LONGITUDINAL  
POTENTIAL DIFFERENCE: CHANGE OF RESISTANCE  
IN A MAGNETIC FIELD

## 1. General

When a conductor,  $C$ , Fig. 80, that carries an electric current,  $I$ , is placed in a magnetic field,  $H$ , the difference in potential between points,  $D$  and  $E$ , along the current, is changed, or the electrical resistance altered. This change in

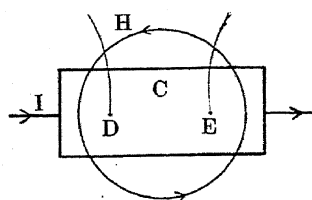


Fig. 80.

resistance in some cases is an increase, while in others it is a decrease.

That a magnetic field brought about a change in resistance in a conductor was first observed by Sir Wm. Thomson <sup>(537)</sup>, 1856, who found that the resistance of *iron* was increased when magnetized longitudinally. In 1857 he obtained a similar result for *nickel*. The resistance of each metal decreased in a transverse field.

The above observations were confirmed by Beetz <sup>(59)</sup>, 1866; Adams and Tomlinson <sup>(9)</sup>, 1875; Chwolson <sup>(119)</sup>, 1877; Auerbach <sup>(32)</sup>, 1878; de Lucchi <sup>(347)</sup>, 1882; and by Tomlinson <sup>(540)</sup>, 1882.

Adams <sup>(9)</sup>, 1875, was the first to point out the fact that the change in resistance,  $\Delta r$ , was proportional to the square of the field strength.

Tomlinson <sup>(540)</sup>, 1882, proposed the following equation for the change of resistance in a magnetic field,  $H$ :

$$\frac{\Delta r}{r} = aH + bI,$$

in which  $a$  and  $b$  are constants depending on the conductor and  $I$  the intensity of magnetization.

Goldhammer <sup>(213)</sup>, 1887, after many experimental determinations, came to the conclusion that the variation of re-

sistance with field strength for *non-magnetic metals* could be represented by the equation:

$$\frac{\Delta r}{r} = AH^2,$$

in which  $A$  is a constant depending on the metal. For *ferro-magnetic metals* he thought the equation should be:

$$\frac{\Delta r}{r} = aI^2,$$

in which  $a$  is a constant, and  $I$  the magnetic moment. In general it has been found that the change of resistance in non-magnetic metals varies as the square of the field strength; while in ferromagnetic metals the change is proportional to the square of the magnetization.

## 2. Change of Resistance in Non-Magnetic Metals

### a. Bismuth in a Transverse Field

The behavior of *bismuth* in a magnetic field is so exceptional that it will be treated separately,

In 1884, Righi <sup>(446)</sup>, Hurion <sup>(270)</sup>, and Leduc <sup>(316)</sup>, independently observed that *bismuth* increased in resistance in a transverse magnetic field.

Ettingshausen <sup>(170)</sup>, 1887, found that the resistance of *bismuth*, *antimony*, and *tellurium* increased in such a field.

Faè <sup>(193)</sup>, 1887, Goldhammer <sup>(213)</sup>, 1887, and Tomlinson <sup>(542)</sup>, 1888, observed that *bismuth* increased in resistance in a transverse field, and also in a longitudinal one.

Van Aubel <sup>(22)</sup>, 1888, measured the change in resistance in *bismuth* in a magnetic field at two temperatures, with the following result.

Bismuth	Temperature	$100 \frac{\Delta r}{r}$
Slowly Cooled . . . . .	0° C.	2.9
" " . . . . .	99.7°	0.415

The increase in resistance accordingly falls off with rise in temperature.

Lenard and Howard <sup>(325)</sup>, 1888, in measuring the change in resistance in *bismuth* in a magnetic field, used spirals of the metal formed by forcing the metal through a small aperture. The spirals were protected on each side by sheets of mica, and the ends of the non-inductively wound spiral were fastened to copper lead wires, Fig. 81.



Fig. 81.

When the plane of the spiral was perpendicular to the magnetic field, the following results were obtained:

$H$	0	2000	6000	10000	12000	14000	16000
$r$	1	1'049	1'217	1'420	1'527	1'634	1'740

When the plane of the spiral was parallel to the lines of force these results were obtained.

$H$	0	7390	10930
$r$	1	1'203	1'302

Lenard and Howard found that the following equation represented the relation between the resistance,  $r$ , and the angle,  $\Phi$ , between the plane of the spiral and the lines of force:

$$r = a + b \sin^2 \Phi - c \sin^2 2\Phi \cdot \cos 2\Phi,$$

where  $a$ ,  $b$ , and  $c$  are constants.

Carpini <sup>(112)</sup>, 1904, proposed for the above relation this equation:

$$r = r_0 + A(1 + \cos 2\Phi),$$

where  $r_0$  is the resistance of the spiral without the field, and  $A$  is a constant.

#### *b. Measurement of the Magnetic Field by Means of a Bismuth Spiral*

Hall <sup>(241)</sup> in 1885 remarked: "It seems probable that a thin slice of *bismuth*, properly prepared and mounted, will come to be a valuable instrument for measuring the intensity

of strong magnetic fields." Leduc <sup>(317, 319)</sup>, 1886, 1888, seems to have been the first to measure the strength of a magnetic field by noting the change in resistance in bismuth. He employed plates of bismuth and narrow glass tubes containing bismuth that had been poured in while molten. The bismuth spiral of Lenard and Howard, Fig. 81, is the form in which bismuth is generally used when the strength of a magnetic field is to be determined, by noting the change in resistance of the metal in the field. The spiral is usually accompanied by a chart containing a curve found by plotting observed changes in resistance of bismuth against known fields, and from this curve is found the strength of a field that produces an observed change in resistance.

Henderson <sup>(256)</sup> called attention to the fact that the *temperature of the bismuth spiral* must be taken into account, since the resistance of bismuth varies with temperature.

A somewhat analogous method of determining the strength of a magnetic field is the so-called "Hall method". A metal, whose Hall effect coefficient  $R$  is accurately known, is placed in an unknown field, and the total difference of potential,  $E$ , measured. Then from the equation,

$$E = \frac{RHI}{d},$$

$H$  may be computed. This method is reliable only if the metal used is non-magnetic and non-crystalline, for only in the case of such metals does  $E$  vary uniformly with  $H$ .

### c. Variation of Resistance of Bismuth with Field and Temperature

The change in resistance in *bismuth* in a magnetic field up to 9000 gauss, and at temperatures from 0° C. to 160° C., was determined by Leduc <sup>(321)</sup>, 1890. He found that the change in resistance could be represented by the equation

$$\frac{\Delta r}{r} = z,$$

where  $z$  is related by the equation  $z^2 + \beta z - \alpha H^2 = 0$ , in



which  $\alpha$  and  $\beta$  are constants. These constants, however, vary with temperature. The above equation did not hold for high fields.

Leduc obtained the following values for  $\alpha$  and  $\beta$  at various temperatures.

	18° C.	44.7°	100°	130.8°	157.5°
$\alpha \times 10^{11}$	221	143	55	33	22
$\beta \times 10^3$	263	275	301	316	342

At the above rate,  $\alpha$  would vanish at 261°, and Leduc assumed that the influence of the magnetic field would disappear at the fusion point of bismuth.

Carpini<sup>(112)</sup>, 1904, arrived at the same expression as Leduc's equation above, while Goldhammer<sup>(213)</sup>, 1887, thought his equation,  $\frac{\Delta r}{r} = aI^2$ , applied also to *bismuth*.

The increase in resistance of *bismuth* in a magnetic field was determined by Drude and Nernst<sup>(154)</sup>, 1890, between temperatures 0° and 290° C., in fields up to 7000 gauss, with the following results.

Temperature	16° C.	100°	223°	290°
$\Delta r$	21.9%	8.0%	0.96%	0.41%

The increase in resistance falls off rapidly with rise in temperature, and seemingly would disappear at the melting point.

Henderson<sup>(256)</sup>, 1894, determined the increase in resistance in *bismuth* from 0° to 80° C., in fields up to 39,000 gauss. The following table gives the results obtained at 18° C.

$H$	$100 \frac{\Delta r}{r}$	$H$	$100 \frac{\Delta r}{r}$
5,830	22.7	28,370	160.9
6,830	29.0	30,090	170.4
8,880	40.7	32,730	184.6
12,500	63.0	35,800	207.0
20,450	116.0	38,900	233.4
26,820	150.8		

The curves in Fig. 82 give the results of measurements made by Henderson for *bismuth* at several temperatures, and show that the resistance increases more rapidly at the lower temperature.

Dewar and Fleming <sup>(145)</sup>, 1896—7, obtained some remarkable results for the increase in the resistance of *bismuth* at very low temperatures. The following table exhibits the change of resistance of electrolytic bismuth wire in transverse fields at various temperatures. The resistances are in absolute units per 1 cm. cube.

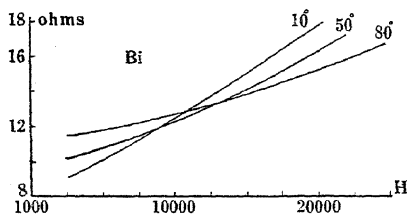


Fig. 82.

Temperature	$H = 0$	$H = 2,450$	$H = 5,500$	$H = 14,200$
	$r_0$	$r$	$r$	$r$
+ 19° C.	116,200	123,500	132,000	187,000
— 79°	78,300	105,000	158,000	284,000
— 185°	41,000	186,000	419,000	1,740,000
— 203°	34,300	283,500	—	—

The resistance not only increases with temperature in zero field, and with the magnetic field, but at the lower temperatures the increase with field is more rapid. At —185° C. the resistance in a field of 14,200 gauss is more than 42 times the resistance in zero field.

Van Everdingen <sup>(182)</sup>, 1898, proposed the following equation for the increase in resistance in a magnetic field:

$$\Delta r = \frac{C_2 M^2}{1 + C_1 \sqrt{M^2}},$$

in which  $C_1$  and  $C_2$  are constants, and  $M$  the magnetic field strength in kilogauss. This equation held fairly well for changes in the resistance of bismuth determined by himself, Henderson <sup>(256)</sup>, Ettingshausen and Nernst <sup>(173)</sup>, and Dewar and Fleming <sup>(145)</sup>.

Van Everdingen <sup>(178)</sup>, 1897, observed that in a *crystal* of *bismuth* the increase in resistance was least when the magnetic field was parallel to the principal axis, and greatest when the field was perpendicular to the axis. The Hall effect in the crystal was found to vary in a similar manner.

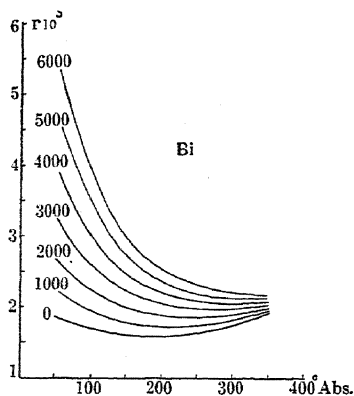


Fig. 83.

In 1900 Van Everdingen <sup>(183)</sup> determined the change in resistance of a pure *bismuth plate* in magnetic fields from 0 to 6000 gauss, and at temperatures from 100° to -182° C. The curves in Fig. 83 exhibit the variation of the resistivity,  $r$ , with temperature in the several fields. The curves are strikingly similar to those determined by Van Everdingen, p. 49, for the variation of the Hall effect in bismuth with temperature.

Du Bois and Wills <sup>(82)</sup>, 1899, measured the resistance of a *bismuth spiral* in a field of 37,500 gauss, at temperatures from 0° to -180° C. The results below show a remarkable increase in resistance at very low temperatures.

Temperature	0° C.	0°	-79°	-115°	-180°
Resistance	No Field I	4	6	10	230

Lloyd <sup>(331)</sup>, 1900, and Yamaguchi <sup>(607)</sup>, 1900, also found that the resistance of *bismuth* in a magnetic field increased greatly at low temperatures, and that in weak fields at low temperatures, the ratio  $m \div \frac{\Delta r}{r}$  was fairly constant, as shown in the table below, due to Yamaguchi. In the above ratio,  $m = Q \cdot H$ , where  $Q$  is the Nernst coefficient, p. 212.

H	$m \div \frac{\Delta r}{r}$			
	- 119° C.	- 30.5°	+ 10°	+ 62°
1293	—	—	9.8	- 29
2650	28	32	19	- 6
4120	27	35	32	+ 6.7
6120	24.5	35	31	+ 27
8650	24	34	31	—

Patterson <sup>(404)</sup>, 1901, observed that the increase in resistance in thin *bismuth films* was much less than in bismuth wires or plates. The table below gives the results, and shows that the increase in resistance falls off rapidly as the thickness of the film is diminished.

Metal	Thickness	H	$\frac{\Delta r}{r} \times 10^4$
Bismuth . . .	$10 \times 10^{-6}$ cm.	$\left\{ \begin{array}{l} 26,200 \\ 27,300 \end{array} \right\}$	$29.0$ (mean)
" . . .	6 "	$\left\{ \begin{array}{l} 26,200 \\ 27,300 \end{array} \right\}$	5.6 "
" . . .	4 "	26,200	1.0

Lownds <sup>(346)</sup>, 1902, measured the change in resistance in a *crystalline* plate of *bismuth*, so cut that the principal axis was parallel to the length of the plate. The change in resistance was measured by determining the fall of potential along the plate when the field was on and when off, the current being so adjusted that it was the same in each case. If  $e_0$  is the fall in potential in zero field, and  $e$  the fall in the field,  $r_0$  the resistance in zero field, and  $r$  the resistance in the field, then, since the current through the plate is constant,

$$i = \frac{e_0}{r_0} = \frac{e}{r}.$$

$$\text{Hence} \quad \frac{r-r_0}{r_0} \quad \text{or} \quad \frac{\Delta r}{r_0} = \frac{e-e_0}{e_0}.$$

The results, both when the current was parallel and perpendicular to the axis, are given in the table below.

	Current    to Axis			Current ⊥ to Axis		
	22°5' C.	-79°	-186°	14° C.	-79°	-186°
<i>H</i>	$\frac{\Delta r}{r}$	$\frac{\Delta r}{r}$	$\frac{\Delta r}{r}$	$\frac{\Delta r}{r}$	$\frac{\Delta r}{r}$	$\frac{\Delta r}{r}$
2120	5.0%	22.5%	33.5%	3.9%	9.6%	6.1%
3120	9.2	37.7	44.2	7.3	14.2	9.7
3500	12.1	43.1	47.4	8.6	15.7	10.6
4980	19.8	62.1	56.5	14.3	21.5	11.4

The increase in resistance is greater along the main axis than perpendicular to that axis, and in general is greater at lower temperatures.

Jewett <sup>(278)</sup>, 1903, determined the change in resistance of *bismuth* in fields up to 15,000 gauss, and at temperatures from -90° to +241° C. He, as others, found that the resistance increased more rapidly at the lower temperatures. Above 200° C. the change in resistance was very small.

Jewett discovered that thin *films of bismuth*, formed by *cathode sputtering*, exhibited no change in resistance in magnetic fields up to 27,000 gauss. This remarkable result was probably due to the non-crystalline structure of the film.

Carpini <sup>(112)</sup>, 1904, investigated the change in resistance of *bismuth wire* in a magnetic field, and found that the change in resistance was given by the following equation:

$$H^2 = \frac{\Delta r}{r} \left( \frac{\Delta r}{r} 4.6318 + 5727.3 \right) 10^4.$$

In a direct current field no lag was discovered, while in an alternating field a lag did exist, and  $\frac{\Delta r}{r}$  was much smaller than in a corresponding direct field.

Blake <sup>(70)</sup>, 1909, measured the change in resistance of *bismuth spirals* in magnetic fields from 600 to 36,600 gauss, and at temperatures from -192° to +183° C. He, as others, found this change in resistance to be a function of both temperature and field strength, and that the increase in resistance is greater at the lower temperatures. At -192° C.

the ratio of the resistance in the field to that out of field,  $\frac{r}{r_0}$ , was in a field of 36,600 gauss about 90.

The change in resistance in *bismuth* in transverse magnetic fields at temperatures from  $17^\circ$  to  $-201^\circ$  C. was measured by Beckman<sup>(53)</sup>, 1912. In weaker fields the curves, showing the increase in resistance with field strength at constant temperature, are convex with respect to the  $H$  axis, while above 12,000 gauss the curves become ascending straight lines. For fields above 12,000 gauss, the following equation held true:

$$\frac{r}{r_0} = aH + b,$$

where  $r$  is the resistance in the magnetic field, and  $r_0$  the resistance in zero field and at  $0^\circ$  C., and  $a$  and  $b$  constants.

The table below gives the change in resistance in *bismuth* at low temperatures as determined by Onnes and Beckman<sup>(395)</sup>, 1912. The increase in resistance at very low temperatures is enormously greater than at ordinary temperature.

*Resistance of Bismuth in Magnetic Field at Low Temperatures.*

$H$	$\frac{r}{r_0}$ at $290^\circ$ Abs.	$\frac{r}{r_0}$ at $20\cdot3^\circ$ Abs.	$\frac{r}{r_0}$ at $15^\circ$ Abs.
0	1'057	0'242	0'216
2,760	1'140	4'73	—
3,850	—	—	8'185
5,540	1'280	13'50	14'35
7,370	1'388	22'50	23'00
9,200	1'495	31'55	33'25
11,850	1'646	46'55	47'90
13,600	1'746	58'20	58'85
15,670	1'868	70'75	72'25
17,080	—	80'85	82'00

The influence of gas pressure on the resistance of *bismuth wire* in and out of a magnetic field was investigated by Brentano<sup>(99)</sup>, in 1915, with the result that in a field of 9000 gauss and under pressures up to 600 atmospheres the

resistance increased steadily, and was greater under pressure in the field than out of it.

The resistance of *pastilles* of compressed finely powdered *bismuth* was measured by Trabacchi<sup>(544)</sup>, 1915. In a field of 5400 gauss the increase was only 4%, while ordinary bismuth under the same conditions increased 20%. Pastilles made from the metal reduced chemically from bismuth salts exhibited practically no increase in resistance in a field of 10,000 gauss. These results must be due to the amorphous condition of the metal.

Corbino and Trabacchi<sup>(532)</sup>, 1915, placed a *circular disk* of *bismuth*, with two concentric circular electrodes, in a transverse magnetic field, and sent a radial current through the disk. Under the action of the field the lines of flow of the current became spirals, and there resulted an increase in the global resistance of the disk. The superposition of the internal currents on the original current increased the energy consumed in the Joule heat effect, and in this way the increase in resistance was accounted for.

Borelius and Lindh<sup>(95)</sup>, 1917, measured the change in resistance in a *bismuth crystal* in a magnetic field, when the field was perpendicular and parallel to the crystal axis. They found that the change varied very much with the direction of the field with respect to the crystal axis, and with the direction of the electric current in the crystal. The results agree in general with those of Everdingen, p. 164.

La Rosa<sup>(458)</sup>, 1919, measured the change in resistance in *bismuth* in the magnetic field, with the results as given below.

*Resistance of Bismuth in a Magnetic Field*

$H$	$\frac{\Delta r}{r}$	$H$	$\frac{\Delta r}{r}$
2100	0.030	5900	0.172
3150	0.061	6600	0.207
3700	0.082	7600	0.253
4500	0.114	9100	0.322
5100	0.136	10100	0.387

Beyond a field of 3000 gauss,  $\frac{\Delta r}{r}$  becomes practically linear with respect to the field.

The magneto-resistance of thin cold *cathode-sputtered films of bismuth* was measured by Becker and Curtiss <sup>(52)</sup>, 1920. At first the films showed no change in resistance even in fields up to 15,000 gauss. After the film had been heated to a maximum of 256° C. and cooled several times the following change in resistance occurred:

$H$	$\frac{\Delta r}{r} \times 100$	$H$	$\frac{\Delta r}{r} \times 100$
1740	0.25	7550	4.4
3400	0.88	8150	5.02
5000	1.88	9500	6.90
6900	3.64		

The value of  $\frac{\Delta r}{r}$  in a given field increased rapidly with temperature up to a maximum temperature to which the film could be heated before melting. Photomicrographs, taken before and after heating, showed that a crystalline structure had been produced by the heating and cooling.

Richtmyer and Curtiss <sup>(435)</sup>, 1920, made a further study of the magneto-resistance of *sputtered bismuth films*. Their work confirmed that of Becker and Curtiss.

Films that were allowed to become heated in the process of sputtering exhibited a change in resistance; while films deposited by intermittent sputtering, and not allowed to become heated, showed practically no change in resistance in fields up to 16,000 gauss.

The foregoing investigations, with those of Jewett (p. 166 and others, point to a profound connection between the crystalline structure of the metal and the magnitude of the several effects herein treated. The orderly arrangement of the atoms and molecules probably plays a more important part in the phenomena than the motion of the free electrons under the influence of the magnetic field.



d. *Bismuth in a Longitudinal Field*

As already noted, p. 159, Faè, Goldhammer, and Tomlinson observed that *bismuth* increased in resistance in a longitudinal field.

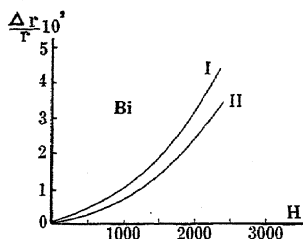


Fig. 84.

Barlow <sup>(44)</sup>, 1903, measured the resistance of *bismuth* in both longitudinal and transverse fields. Curve I, Fig. 84, represents the change in resistance in a longitudinal field, while Curve II gives the change in a transverse field.

The rate of change and the magnitude are nearly the same in each field.

e. *Other Non-Magnetic Metals in a Magnetic Field*

In 1887, Faè <sup>(192)</sup> discovered that *antimony* increased in resistance in both transverse and longitudinal fields.

Goldhammer <sup>(213)</sup>, 1887, found that *antimony* and also *tellurium* increased in resistance in both types of field.

The measurements by Lenard <sup>(324)</sup>, 1890, of the change in resistance of *antimony* and *tellurium* in a transverse field for direct and alternating currents are given in the table below.

Antimony			Tellurium		
<i>H</i>	Resistance D. C.	Resistance A. C.	<i>H</i>	Resistance D. C.	Resistance A. C.
0	1.000	0.9915	0	1.000	0.9926
6620	1.012	1.002	11450	1.0016	0.9983

Patterson <sup>(404, 6)</sup>, 1901—2, measured the change in resistance in a number of *non-magnetic metals* in both transverse and longitudinal fields. In all metals examined the resistance increased in both types of fields, as shown by the tables below. In all cases  $\frac{\Delta r}{r}$  varied as the square of the field strength.

*Magneto-Resistance in Transverse Field*

Metal	H	$\frac{\Delta r}{r}$	Metal	H	$\frac{\Delta r}{r}$
Cu . . . . .	27,650	$1.96 \times 10^{-4}$	Pt . . . . .	28,600	$0.44 \times 10^{-4}$
Hg . . . . .	24,900	3.18 "	Sn . . . . .	28,600	1.84 "
Cd . . . . .	28,600	20.86 "	Ag . . . . .	28,600	1.76 "
Zn . . . . .	28,600	5.98 "	C . . . . .	24,400	2.73 "
Au . . . . .	28,600	2.93 "			

*Magneto-Resistance in Longitudinal Field*

Metal	H	$\frac{\Delta r}{r}$	Metal	H	$\frac{\Delta r}{r}$
Ag . . . . .	22,700	$0.98 \times 10^{-4}$	Zn . . . . .	22,700	$3.09 \times 10^{-4}$
" . . . . .	20,300	0.79 "	" . . . . .	20,300	2.46 "
Sn . . . . .	22,700	0.93 "	Cu . . . . .	23,500	1.80 "
" . . . . .	20,300	0.74 "	" . . . . .	20,800	1.54 "
German Silver	26,600	No change			

The resistance of *antimony* in a transverse field was found by Barlow <sup>(44)</sup>, 1903, to increase more rapidly at higher fields, as shown in Fig. 85.

The change in resistance in *antimony* in a transverse field was also determined by Zahn <sup>(609)</sup>, 1904, with the following results.

Metal	H	$\frac{\Delta r}{r}$
Sb . . . . .	6,290	+ 0.0095
" . . . . .	10,500	+ 0.0215

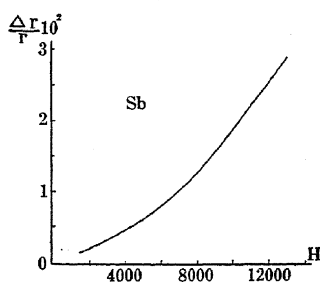


Fig. 85.

The curves in Fig. 86 exhibit the variation of the resistance of a number of *non-magnetic metals* in a transverse field, as determined by Grunmach and Weidert <sup>(230)</sup>, 1906.

The ordinates of the curves are in terms of  $\frac{\Delta r}{r}$ .

In each case the resistance increases at first slowly, then more rapidly, and finally the change becomes linear.

If the above metals, paramagnetic and diamagnetic, are arranged according to the magnitude of the increase in re-

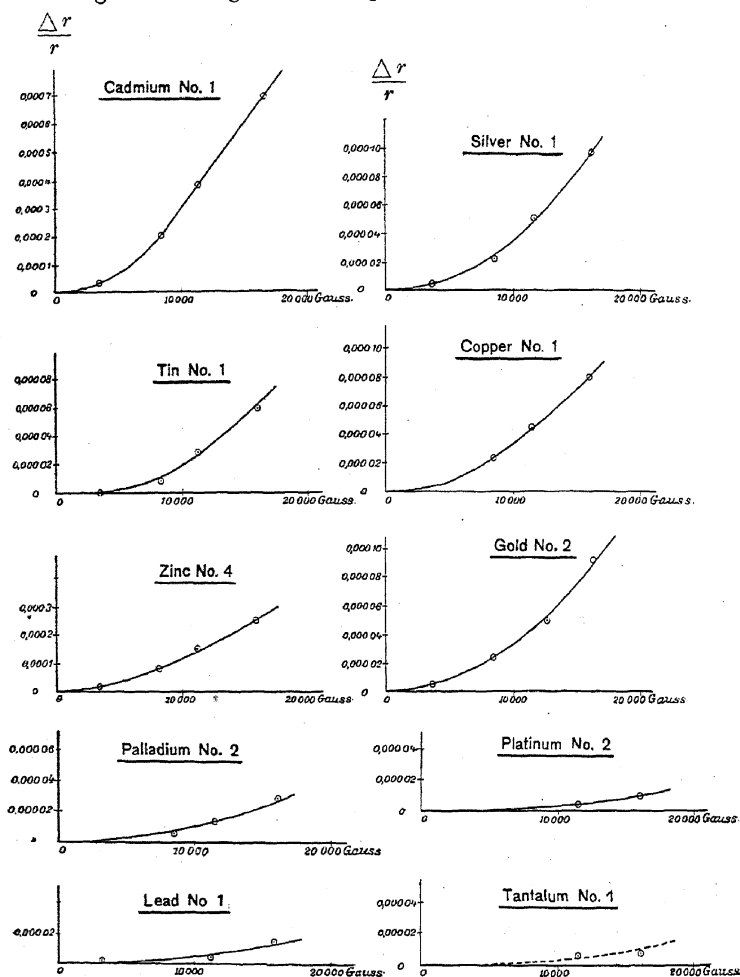


Fig. 86.

stance in a transverse field, the following decreasing series will be obtained: *Bi, Cd, Zn, Ag, Au, Cu, Sn, Pd, Pb, Pt, Ta.*

Dagostino<sup>(140)</sup>, 1908, also found that the following metals

increased in resistance in a transverse magnetic field: *Pt, Ag, Bi, Cd, Au, Zn, Mg, Pd, invar*, and *manganin*.

The resistance of *selenium* in a transverse magnetic field was found by Amaduzzi <sup>(25)</sup>, 1909, to increase. He pointed out that metals, arranged according to the magnitude of the change of resistance in a magnetic field, will have the same order as when arranged according to the magnitude of their Hall effects. Such a series is: *Bi, Ni, Fe, Mg, Zn, Au, Pt*. But *Cd* and *Pd* are apparent exceptions.

The change in resistance of *zinc, cadmium* and *graphite* was measured by Laws <sup>(31)</sup>, 1910, in a transverse field at several temperatures. In all three cases  $\Delta r$  varied as  $H^2$ . The table below gives the values of  $A$  from the equation,  $\frac{\Delta r}{r} = AH^2$ .

Cadmium	98° C.	55°	16°	— 186° C.
$A \times 10^{12}$	0.99	1.60	2.65	51
Zinc	—	56.1°	18.5°	— 186°
$A \times 10^{12}$	—	0.54	0.87	18.0
Graphite	—	—	15.5°	— 186°
$A \times 10^{12}$	—	—	78.2	225

The next table gives the determination of  $A$  for several metals made by Heaps <sup>(248)</sup>, 1912, in both transverse and longitudinal fields.

Metal	Transverse $H$	Longitudinal $H$
Te . . . . .	$105.0 \times 10^{-12}$	$27.7 \times 10^{-12}$
Cd . . . . .	2.88 „	1.77 „
Zn . . . . .	1.36 „	0.82 „
Au . . . . .	0.38 „	0.32 „

Heaps also measured the resistance, in a transverse field, of *pyrite, galena, magnetite*, and *molybdenite*, with the following results.

Substance	$H$	$\frac{\Delta r}{r}$
Galena . . . . .	10,000	$+5 \times 10^{-6}$ to $+4.5 \times 10^{-4}$
Pyrite . . . . .	"	$< 2 \times 10^{-5}$
Molybdenite . . .	"	$-3.7 \times 10^{-4}$
Magnetite . . . .	"	Varied along different axes

The following values of  $A$ , from equation  $\frac{\Delta r}{r} = AH^2$ , for *cadmium* and *zinc*, determined by several observers, show a remarkable agreement.

Metal	Heaps	Laws	Patterson	Grunmach and Weidert
Cadmium	$28.8 \times 10^{-12}$	$26.5 \times 10^{-12}$	$28.2 \times 10^{-12}$	$27.3 \times 10^{-12}$
Zinc . . .	13.6 "	8.7 "	8.7 "	11.0 "

The magneto-resistance of *Ceylon graphite* was measured by Roberts <sup>(454)</sup>, 1912, in a transverse field, at temperatures from  $-179^\circ \text{C.}$  to  $+179^\circ \text{C.}$  At  $18^\circ \text{C.}$ , in a field of 20,000 gauss the resistance was from 300% to 500% of that in zero field. At liquid air temperature, in a field of 38,800 gauss the increase was 9300%. At  $18^\circ \text{C.}$ , in the same field, the increase was 1250%.

Onnes and Beckman <sup>(396)</sup>, 1912, measured the magneto-resistance of *solid mercury*, in a field of 10,000 gauss, with this result:  $\frac{\Delta r}{r} = 1.5 \times 10^{-3}$  at  $20.3^\circ \text{Abs.}$ , and  $\frac{\Delta r}{r} = 6 \times 10^{-3}$  at  $14.5^\circ \text{Abs.}$

Onnes and Beckman <sup>(395)</sup>, 1912, determined the change in resistance of *gold*, *copper*, and *palladium* at very low temperatures, in a transverse magnetic field. The following table gives the results in terms of  $\frac{w'}{w_0}$ , where  $w'$  is the resistance in the field, at the given temperature, and  $w_0$  the resistance at  $0^\circ \text{C.}$  in zero field.

$H$	Temperature	Gold	Copper	Palladium
10,000	20.3° Abs.	1.017	1.14	1.0015
"	14.5° "	—	1.10	—

The above changes in resistance are much larger than those found at ordinary temperatures.

Onnes <sup>(394)</sup>, 1914, observed that pure *tin* and *lead* in the superconducting state at very low temperatures, when placed in a magnetic field, resumed a considerable resistance, whereas outside of the field, at these extremely low temperatures, their resistance was practically zero. At liquid helium temperatures, 2° Abs. and 4.25° Abs.,  $\frac{\Delta R}{R} \cdot 10^3$

*tin* exhibited a sudden increase in resistance at a threshold value of about 200 gauss, while *lead* at the same temperatures showed a sudden increase in resistance between 500 and 700 gauss. The curves in Fig. 87, show this sudden

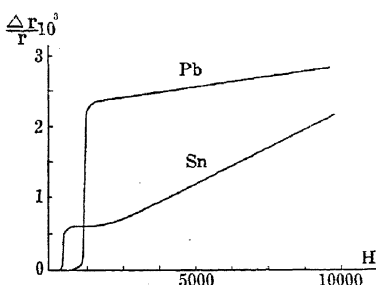


Fig. 87.

rise in resistance at 2° Abs., followed by a gradual increase with rise in field.

The magnetic field has the same effect on the metals as heating them. Onnes remarked: "One would be inclined to assume that an energy of rotation determined by the magnetic field might be simply added to the energy of the irregular molecular motion. If, in the production of the obstructions which determine the resistance we have to do with dissociations in the sense that movements of electrons in certain paths become unstable at a definite temperature, the magnetic centrifugal force might make their motions one-sidedly unstable at another temperature." "In fact, if it were once proved—to use an image already introduced into my paper for the Congress in Chicago <sup>(393)</sup>—that the vibrators

which cause the resistance can only be set in motion when the stream of electrons passes them with sufficient rapidity, then it would not be surprising that the magnetic resistance does not arise until the rapidity of the circulating motions of the electrons is great enough to carry the atoms with it and set them in rotation, by which they can then disturb the regular motion of the electrons."

Silsbee <sup>(482)</sup>, 1916, in discussing the results of Onnes on superconductivity, stated that, since a metal in the superconducting state resumes its normal resistance when carrying a critical current, and also regains its normal resistance in a critical magnetic field, the threshold current density is that at which the critical magnetic field is equal to the field due to the current itself. That is, that in each case the return of the normal resistance is due to the action of a magnetic field on the electrons without and within the atom, and is not due to heating effects and other internal causes.

Onnes and Hof <sup>(398)</sup>, 1914, determined the change in resistance in *lead*, *tin*, *cadmium*, and *graphite*, at about  $20^{\circ}$  Abs., and in *platinum* and *graphite* at  $4.25^{\circ}$  Abs., in fields up to 12,000 gauss. In the case of lead and tin the increase in resistance was practically the same in both transverse and longitudinal fields, while for cadmium and platinum the transverse field gave the larger increase. The increase in resistance in all cases was greater than at ordinary temperatures, and the rate of increase diminished in higher fields.

The change in resistance in the magnetic field in *antimony* plates split along the basic cleavage was determined by De Haas <sup>(232)</sup>, 1914, both when the crystallographic axis was perpendicular and when parallel to the lines of magnetic force.

The table below gives the results of the measurements.

The change in resistance in a magnetic field in Ceylon, Mexican, and American *graphite* was studied by Washburn <sup>(571)</sup>, 1915. The electric current was caused to flow parallel to the principal axis of the crystal. The angle,  $\Phi$ , between this

*Resistance of Antimony: Axis Perpendicular to Magnetic Field*

18° C.		— 188° C.	
$H$	$\frac{r}{r_0}$	$H$	$\frac{r}{r_0}$
21,400	1'101	6,600	1'239
23,200	1'114	12,400	1'620
28,300	1'154	19,800	2'210
29,980	1'167	30,900	3'396
		43,300	5'195

*Resistance of Antimony: Axis Parallel to Magnetic Field*

18° C.		— 188° C.	
$H$	$\frac{r}{r_0}$	$H$	$\frac{r}{r_0}$
21,400	1'039	6,700	1'121
23,200	1'047	12,200	1'310
28,300	1'071	19,800	1'630
29,980	1'079	30,100	2'200
32,000	1'091	43,300	3'060

axis and the magnetic lines of force was varied from  $0^\circ$  to  $90^\circ$ . It was found that the change in resistance could be represented by the equation,

$$\frac{\Delta r}{r} = a + b \cos \Phi.$$

In a field of 30,000 gauss the increase in resistance was about 80%, when  $\Phi$  was zero. The value of  $\frac{\Delta r}{r}$  was between 6 and 7 times greater when  $\Phi$  was  $0^\circ$  than when  $\Phi$  was  $90^\circ$ .

Königsberger and Gottstein<sup>(297)</sup>, 1915, in their work on the change of resistance in variable conductors and metals, give the following results for the change of resistance in transverse fields. The results are in terms of  $A = \frac{\Delta r}{r H^2}$ .

Silver	Copper	Graphite	Zinc	Palladium	Silicon
$+2.6 \times 10^{-13}$	$+2.6 \times 10^{-13}$	$+6.5 \times 10^{-9}$	$+8.7 \times 10^{-13}$	$+1.1 \times 10^{-13}$	$+2.6 \times 10^{-12}$



The variation of the resistance of *tellurium*, at temperatures from  $0^{\circ}$  to  $400^{\circ}$  C., in transverse magnetic fields up to 15,000 gauss, was determined by Wold<sup>(604)</sup>, 1916. The increase in resistance was found to be closely proportional to the square of the field strength. The increase in resistance, in a constant field of 7,580 gauss, at different temperatures, is given by the curve in Fig. 88. The increase in resistance falls off with rise in temperature, and approaches zero above  $400^{\circ}$  C.

*Tellurium*, according to Königsberger and Schilling<sup>(298)</sup>, is a conductor of the second class, in which the number of

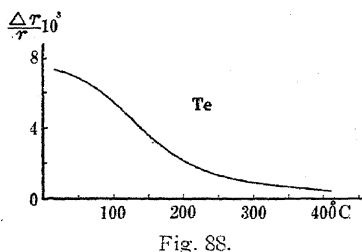


Fig. 88.

free electrons increases with temperature, and hence outside the magnetic field, the resistance decreases with rise in temperature. After a certain temperature is reached, the increase in free electrons ceases, and then the resist-

ance should increase with further rise in temperature. Tellurium up to the highest temperature employed showed no evidence of such an increase in resistance.

In conductors of the first class, such as the ordinary conducting metals, the number of free electrons is supposed by Königsberger and Schilling to be constant, and the increase in resistance with rise in temperature is due to a greater collision activity of the electrons. In conductors of the second class, such as *tellurium*, *silicon*, *carbon*, the increase in collision activity with rise in temperature occurs also, but is more than offset by the increase in the free electrons that carry the current. When the temperature is reached at which no more electrons are set free, the increased collision activity produces an increase in resistance. Königsberger and Schilling found that the resistance of *titanium*, *pyrite*, and *magnetite* decreased with rise in temperature, reached a minimum, and then increased with further rise in temperature.

Further study of the conductors of the second class would seem desirable.

The change of resistance in *graphite* and *cadmium* was determined by Heaps <sup>(251)</sup>, 1917, both in transverse and longitudinal magnetic fields, up to 8000 gauss, at about 26° C. Graphite, in the form of rods made from compressed powders of different fineness, and in the form of natural laminated crystals, was examined. The increase in resistance was greater for the coarser powder, and very much greater in the crystal specimens. In all of the graphite studied and in the case of cadmium, the increase in resistance in the transverse field was greater than that in the longitudinal. These results are in accord with the theory of Heaps, p. 205.

Beckman <sup>(55)</sup>, 1917, determined the change in resistance in several specimens of *tellurium* in a transverse magnetic field at 17° C., and found that  $\Delta r$  varied as  $H^2$ . The table below gives the values of  $A = \frac{\Delta r}{r} \div H^2$ .

*Resistance of Tellurium in a Transverse Field*

Tellurium	A	Tellurium	A
No. 7a	$3.53 \times 10^{-11}$	No. 9	$4.45 \times 10^{-11}$
8a	3.59	11	6.52
8c	4.18	12	6.88
8d	4.01	14	8.38

### *Summary.*

In general, the magneto-resistance of *non-magnetic metals*, in both transverse and longitudinal fields, is an *increase*. The rate of increase is greater in higher fields and at lower temperatures. The change in resistance is proportional to the square of the field strength, and depends in magnitude upon the direction with respect to the crystallographic axis.

### 3. Change of Resistance in Ferromagnetic Metals

As already stated, p. 158, Sir Wm. Thomson found that the electrical resistance of *iron* and *nickel* was increased when

magnetized longitudinally, and decreased when magnetized transversely. Beetz <sup>(59)</sup>, 1866, observed that *iron* increased in resistance in a longitudinal field. In a transverse field he found no change in resistance. The increase in resistance in a longitudinal field he considered a consequence of the change in length of the iron in the field.

The following table gives some of the earliest quantitative results, as determined by Tomlinson <sup>(540)</sup>, 1882, in a longitudinal field.

Metal	Condition	$\frac{\Delta r}{r} \div H$	Metal	Condition	$\frac{\Delta r}{r} \div H$
Iron . .	Annealed	$+2335 \times 10^{-8}$	Nickel .	Annealed	$+8070 \times 10^{-8}$
Steel . .	"	$+1500$ "	" .	Unannealed	$+4343$ "
" .	Unannealed	$+1137$ "	Cobalt .	"	$+628$ "
" .	Very hard	$+70$ "			

Faè <sup>(192)</sup>, 1887, discovered that *cobalt* also decreased in resistance in a transverse field and increased in resistance in a longitudinal field.

Goldhammer <sup>(213)</sup>, 1887—9, studied the magneto-resistance of *iron*, *nickel*, and *cobalt* plates, made by electroplating on platinized glass. In all three metals the resistance increased in the longitudinal and decreased in the transverse field. In both fields the change in resistance approached saturation values in higher fields.

According to Von Wyss <sup>(606)</sup>, 1889, the change in resistance in *iron* in a magnetic field is proportional to the magnetic moment.

Adams <sup>(9)</sup>, Auerbach <sup>(32)</sup>, and Chwolson <sup>(119)</sup> confirmed the observations of the above investigators as to the change in resistance of *iron* in a magnetic field.

Garbosso <sup>(210)</sup>, 1891, seems to have been among the first to note an apparent increase in the resistance of *iron* and *nickel* in a transverse field. The increase in *iron* reached a maximum at about 3000 gauss, while in *nickel* the maximum occurred at 2000 gauss.

Beattie <sup>(51)</sup>, 1898, investigated the magneto-resistance of *iron*, *nickel* and *cobalt* films on platinized glass, in a transverse field, in order to determine the relation between the change in resistance and the intensity of magnetization and the Hall effect.

In fields up to 16,000 gauss the magneto-resistance of *nickel* and *cobalt* was found to be proportional to the square of the intensity of magnetization. In the case of *iron* the change in resistance did not seem to follow this law exactly.

The curves in Fig. 89 show the change in resistance of *nickel* and *cobalt* in a transverse field. One specimen of nickel gave an apparent positive maximum.

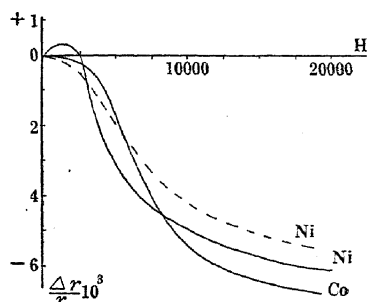


Fig. 89.

For a probable explanation of this, see p. 187.

The table below gives an apparent relation between the Hall effect and the magneto-resistance of *cobalt*.

$H$	$\frac{R}{\sqrt{\Delta r}}$	$\frac{\Delta r}{r}$	$H$	$\frac{R}{\sqrt{\Delta r}}$	$\frac{\Delta r}{r}$
2,400	10.2	$36.44 \times 10^{-5}$	10,300	10.1	$47.44 \times 10^{-5}$
5,100	10.2	92.75 "	11,700	10.2	608.2 "
6,000	10.5	175.6 "	13,800	10.5	644.0 "
8,500	10.3	245.8 "	16,160	10.5	679.6 "

The above relation holds equally well for *nickel*. Since the Hall effect is proportional to the intensity of magnetization in these metals, the change in resistance must therefore be proportional to the square of the intensity.

Gray and Jones <sup>(220)</sup>, 1900, determined the change in resistance in *iron wire*, so wound in two coils that one was in a longitudinal field, while the other was transverse to the same field.

The difference between the change in resistance when magnetized longitudinally and transversely was represented by

$$\Delta\Phi = \frac{\Delta P}{P} - \frac{\Delta Q}{Q},$$

where  $P$  and  $Q$  were the respective resistances of the wires out of the field. If  $\Delta\Phi$  were plotted against  $H$ , the curve became less and less steep as the field increased, and seemed to approach a saturation value.

But if  $\Delta\Phi$  were plotted against  $B^4$ , or  $I^4$ , the curve was practically a straight line, where  $B$  is the magnetic induction,

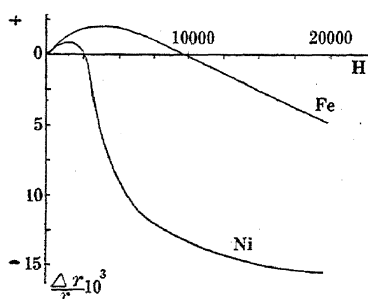


Fig. 90.

and  $I$  the intensity of magnetization. The investigators found that  $\Delta\Phi$  was given by the equation,

$$\Delta\Phi = aI^4,$$

where  $a$  is a constant depending on the metal.

The change of resistance of *iron* and *nickel* in longitudinal and transverse

fields was examined by Barlow <sup>(43)</sup>, 1902, who found that the change in resistance was best represented by the equation:  $\frac{\Delta r}{r} = aI^2 + bI^4 + cI^6$ , in which  $I$  is the intensity of magnetization, and  $a, b, c$  are constants. A hysteresis effect was noted in the magneto-resistance of the two metals.

The curves in Fig. 90 give the variation of the resistance of *nickel* and *iron* in a transverse field. In each case a positive maximum is followed by a decrease that becomes linear in higher fields.

Knott <sup>(288—290)</sup>, 1902—1914, investigated the change in resistance in *iron*, *steel*, and *nickel* in both transverse and longitudinal fields, at temperatures from  $-12^\circ$  to  $350^\circ$  C.

In longitudinal fields up to 34 gauss, the resistance of *nickel* increased, but increased less rapidly as the temperature was increased. The increase in resistance in the field ap-

proached zero at about  $350^{\circ}$  C., the temperature at which nickel loses its strong magnetic properties.

In weak transverse fields up to 60 gauss, and at constant temperature, between  $-12^{\circ}$  and  $93^{\circ}$  C., the resistance in *nickel* increased with field strength.

In constant transverse fields between 2000 and 4000 gauss the decrease in resistance in *nickel* fell off with rise in temperature, until a minimum was reached at about  $295^{\circ}$  C. This was followed by an increase to a maximum at about  $310^{\circ}$  C., then occurred a rapid falling off, and an approach to zero in the neighborhood of  $350^{\circ}$  C. This unusual variation at the higher temperatures was attributed to the fact that a marked change occurs in the Thomson effect at these temperatures, and that the metal is about to lose its susceptibility, and hence is in a very sensitive state.

Knott and Ross <sup>(291)</sup>, 1903, found that in a transverse field the change in resistance of *nickel* reached a positive maximum at 900 gauss, changed sign at 1300 gauss, and then continued to decrease in higher fields.

Williams, <sup>(589-90)</sup> 1903—5, examined the influence of stress and temperature on the magneto-resistance of *iron*, *nickel*, and *nickel-steel*, at temperatures from  $0^{\circ}$  to  $665^{\circ}$  C.

In a longitudinal field up to 900 gauss, the increase for *nickel* diminished with rise in temperature, and from  $0^{\circ}$  to  $260^{\circ}$  C. approached a saturation value. Between  $295^{\circ}$  and  $345^{\circ}$  C.,  $\Delta r$  first increased with the field, then fell off, and between  $328^{\circ}$  and  $345^{\circ}$  C. changed sign.

*Iron* behaved in a similar manner, except that no reversal in sign was observed within the limits of temperature and field employed.

In a transverse field  $\Delta r$  for both *iron* and *nickel* reached a maximum, decreased, and then changed sign.

According to Dongier <sup>(146)</sup>, 1903,  $\frac{\Delta r}{r}$  for *nickel* reached a maximum in a field of 1500 gauss.

The following changes in the resistance of *iron* and *nickel* were measured by Zahn <sup>(609)</sup>, 1904, in a transverse field.

Metal	$H$	$\frac{\Delta r}{r}$
Iron . . . . .	6,290	-0.0008
Nickel . . . . .	6,290	-0.0056
" . . . . .	10,840	-0.0067

Grunmach and Weidert, <sup>(230)</sup> 1906, measured the change in resistance of *iron*, *nickel* and *cobalt* in transverse fields, at about 20° C.

The curves in Fig. 91, 92, 93, in which the ordinates are  $\frac{\Delta r}{r}$ , exhibit the variation of the resistance with field strength.

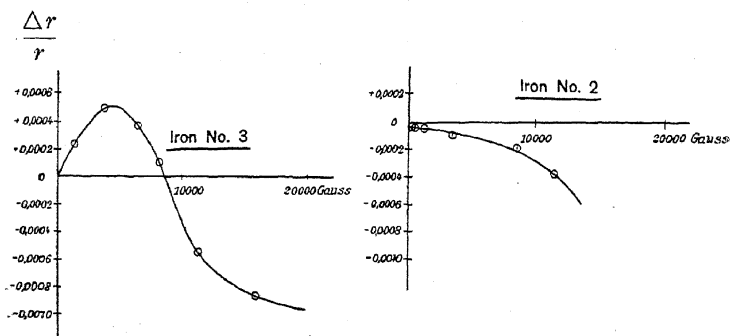


Fig. 91.

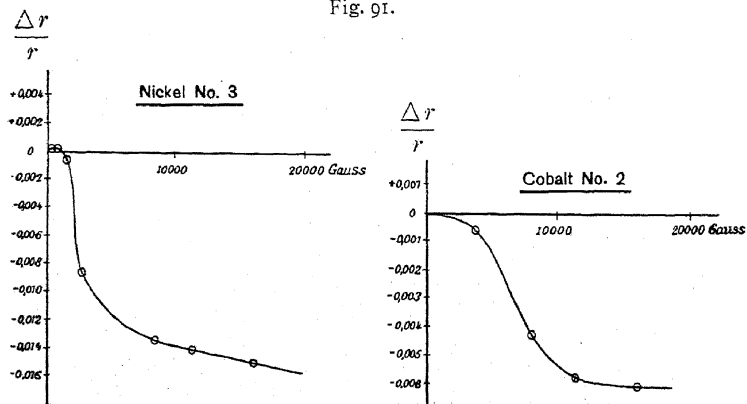


Fig. 92.

Fig. 93.

*Iron* No. 3 and *nickel* No. 3 showed an initial increase to a maximum, followed by a decrease in higher fields. Consult p. 187.

The decrease in resistance in all three metals reached limiting values in high fields.

Dagostino <sup>(140)</sup>, 1908, however, observed that both *iron* and *nickel* exhibited only a decrease in resistance in a transverse field, and that a limiting value was approached in a field of 6000 gauss.

Dumermuth <sup>(156)</sup>, 1908, on the other hand, noted an increase in the resistance of *iron* in weak transverse fields up to 160 gauss. No measurements were made in higher fields and no maximum or reversal was observed.

Blake <sup>(70)</sup>, 1909, determined the change in resistance of *nickel* in transverse fields up to 35,000 gauss, and at temperatures from  $-190^{\circ}\text{C.}$  to  $+182^{\circ}\text{C.}$  The resistance increased in weak fields, reached a maximum, then decreased and changed sign, and at higher fields seemed to approach limiting values. The decrease in resistance in higher fields was greater at the higher temperatures.

Owen <sup>(399)</sup>, 1911, and Heaps <sup>(248)</sup>, 1911, each observed that the resistance of *iron* and *nickel* in a longitudinal field exhibited an increase, that reached a maximum, which was followed by a slight falling off; but there was no reversal of sign, as they found in transverse fields.

Onnes and Beckman <sup>(396)</sup>, 1912, measured the magneto-resistance of *iron* and *nickel* in a transverse field, at  $15^{\circ}\text{C.}$  and at  $-252.7^{\circ}\text{C.}$

The curve I in Fig. 94 shows that  $\frac{\Delta r}{r}$  for *iron* at  $15^{\circ}\text{C.}$  reaches a positive maximum, and then

changes sign; while curve II indicates that at  $-252.7^{\circ}\text{C.}$   $\frac{\Delta r}{r}$  reaches a negative maximum followed by a change of sign.

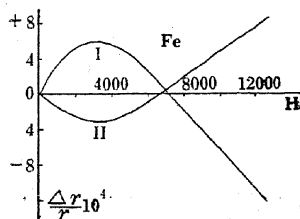


Fig. 94.



*Nickel* in fields below 3000 gauss exhibited a small positive maximum, followed by an change of sign in higher fields, but no negative maximum was observed, as was the case in iron.

The resistance of *monel*, *nichrome* and *nickel* was found by Smith <sup>(490)</sup>, 1913, to decrease in a transverse field. In a longitudinal field the resistance of these metals increased rapidly at first, and in higher fields approachend saturation values.

Heising <sup>(255)</sup>, 1914, could observe no initial increase in resistance in *electrolytic iron* in a transverse field. The resistance began at once to decrease in weak fields and to continue to decrease in higher fields.

In longitudinal fields the increase in resistance of the *electrolytic iron* was slightly greater than that for ordinary iron.

A most careful and important study of the change of resistance of *nickel* in transverse and longitudinal fields was made by Jones and Malam <sup>(280)</sup>, 1914. The specimen of nickel was

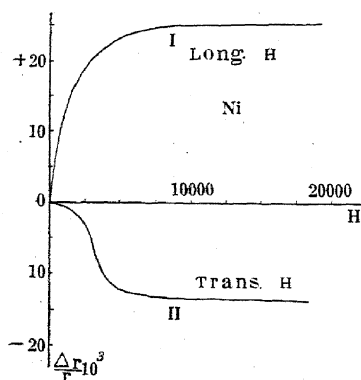


Fig. 95.

first placed in a longitudinal field, and the change in the resistance with field strength was measured. The specimen was then turned so that it made successively increasing angles with the lines of force. When the angle reached  $90^\circ$ , the specimen was in a transverse field. The variation of the resistance at the successive angles was determined.

The curve I in Fig. 95 shows that the resistance in a longitudinal field increases rapidly at first, and then reaches a saturation value. While the curve II shows that in weak transverse fields the decrease is not so rapid at first, but in higher fields increases more rapidly, and then approaches a limiting value.

The uppermost curve in Fig. 96 gives the variation of the resistance in *nickel* in a longitudinal field. The lowest curve is for a transverse field, while the intermediate curves show the variation of the resistance for the successive orientations of the metal. The longitudinal curve changes gradually into the transverse.

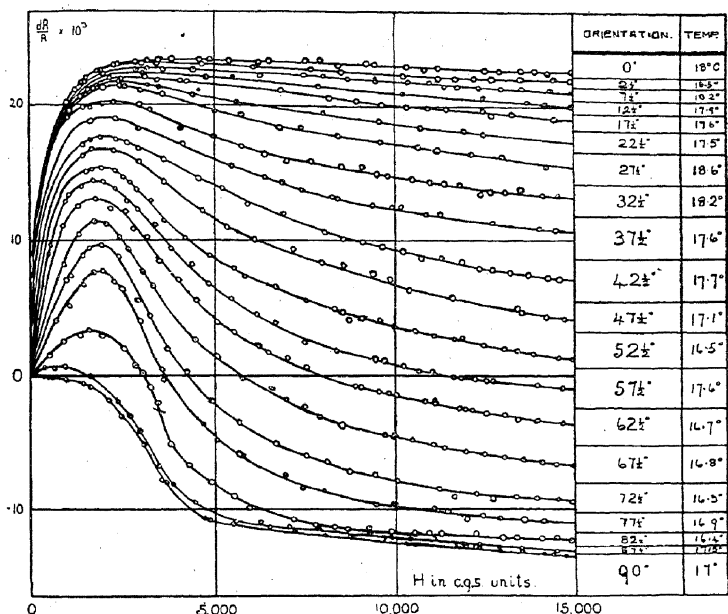


Fig. 96.

In the above variation of the magneto-resistance of *nickel* in the two types of field and in the intermediate fields, we probably have an explanation why Beattie <sup>(51)</sup>, Barlow <sup>(43)</sup>, Blake <sup>(70)</sup>, Grunmach and Weidert <sup>(230)</sup>, and others found an initial increase in resistance of the ferromagnetic metals in a transverse field.

If the metal plates, employed by these observers, made an angle with the lines of force slightly less than 90°, or if the field was not uniform and some of the lines were oblique

to the plate, then in the so-called transverse field a longitudinal component would be present; and since, according to Fig. 95, in weak longitudinal fields the resistance increases extremely rapidly, the positive longitudinal component may predominate over the negative transverse component, which increases very slowly at first in weak fields. In this way the initial positive maximum in weak transverse fields may be accounted for. Note the small positive maximum in the next to the last curve in Fig. 96, when the metal made an angle of  $87\frac{1}{2}^\circ$  with the lines of force.

Zahn and Schmidt <sup>(616)</sup>, 1907, thought they detected in *Heusler alloy* a small increase in resistance in strong fields, namely:

$$\frac{\Delta r}{r} = 0.0007 \text{ (approx.)}$$

However, Heaps <sup>(248)</sup>, 1911, observed a decrease in the resistance in the *Heusler alloy* in both transverse and longitudinal fields. The  $\frac{\Delta r}{r}: H$  curve was convex for the transverse and concave for the longitudinal field.

In transverse fields up to 30,000 gauss, Bonazzi <sup>(89)</sup>, 1914, found that the resistance of the *Heusler alloy* decreased as the field increased, and in higher fields approached a limiting value. In a field of 30,000 gauss he found  $\frac{\Delta r}{r} = -5.4 \times 10^{-3}$ .

### Summary.

The *ferromagnetic metals*, in general, increase in resistance in a *longitudinal* field, rapidly at first, followed by an approach to a limiting value at magnetic saturation.

In a *transverse* field the resistance of *ferromagnetic metals* decreases, slowly at first, then rapidly, followed by an approach to a saturation value. The apparent initial positive maximum in a transverse field seems due to lack of perfect orientation, or of uniform field. See Figs. 95 and 96.

The change in resistance in each field is proportional to the square of the intensity of magnetization, and not to that of the field strength.

#### 4. Magneto-Resistance in Combined Fields.

Knott <sup>(290)</sup>, 1913—14, examined the change in resistance of a *nickel* strip in simultaneous longitudinal and transverse fields. While one field was kept unchanged in direction, the other was reversed, and *vice versa*.

When a reversed longitudinal field,  $H_l$ , was superposed on a steady transverse field,  $H_t$ , the increase in resistance diminished as  $H_l$  was increased, and beyond a critical value changed sign.

A reversed  $H_l$  superposed on a steady  $H_t$  caused a greater decrease in resistance, as  $H_l$  was increased.

In separate fields, the change in resistance of a strip of *nickel* 2.1 cm. wide was greater in a longitudinal field. In strips 2, 4, and 8 cm. wide, the resistances in equal transverse fields were to each other as 22.5:47:62. Knott thought that if the strip were as wide as long, the change in resistance in each type of field would be of the same magnitude, but of opposite sign.

The changes in resistance in *iron* in combined fields were similar to those in *nickel*.

#### 5. Magnetostriction and Magneto-Resistance.

In the case of *ferromagnetic metals*, at least, there seems to exist some intimate relation between the change in resistance in a magnetic field and magnetostriction. The change in dimension of the metal probably does not account for the total change in resistance, yet it must have a part in this change.

Joule <sup>(281)</sup>, Barrett <sup>(45)</sup>, Tomlinson <sup>(540)</sup>, Bidwell <sup>(65)</sup>, Taylor Jones <sup>(279)</sup>, Barlow <sup>(43)</sup>, Nagaoka <sup>(384—85)</sup>, Honda and Shimizu <sup>(265)</sup>, Shaw and Laws <sup>(481)</sup>, and others have shown that in

general *nickel* and *iron* decrease in length in a longitudinal magnetic field.

W. E. Williams <sup>(588-90)</sup>, 1902-5, investigated the relation between the change in length and the change in resistance in *nickel* in a longitudinal field. The curves, that represent the increase in resistance and the decrease in length of the piece of nickel wire, with the increase in field strength, were strikingly alike, and similar to those of Heaps in Fig. 97 below:

Jenkins <sup>(277)</sup>, 1914, examined the change of resistance and length in several specimens of *nickel* in a longitudinal field, and found that for fields below 100 gauss  $\frac{\Delta r}{r}$  was proportional

to  $\sqrt{\frac{\Delta l}{l}}$ .

Starting with the usual theory of electronic conduction in metals, Heaps <sup>(249)</sup>, 1915, deduced the relation

$$\frac{\Delta r}{r} = C \frac{\Delta l}{l},$$

where  $C$  is a constant depending on the metal. Heaps found that both *nickel* and *iron* contracted in a longitudinal magnetic field, while the resistance of each increased.

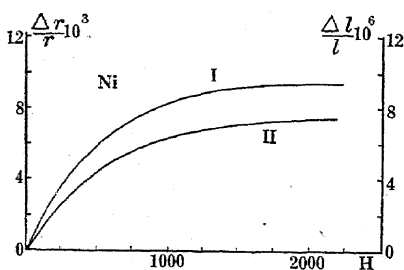


Fig. 97.

The curves I and II in Fig. 97 indicate a close relationship between  $\frac{\Delta r}{r}$  and  $\frac{\Delta l}{l}$  for nickel. The curves for iron showed a like correspondence.

In a transverse field *iron* was found to contract, while *nickel* expanded. The  $\frac{\Delta r}{r}$  and  $\frac{\Delta l}{l}$  curves for *nickel* resemble each other very closely, while the curves for *iron* are not similar. This latter discrepancy would seem to demand further investigation.

It is possible that the Wiedemann <sup>(581)</sup> twist and the Villari <sup>(556)</sup> reversal in iron may play a part in the discrepancy in the variation of  $\frac{\Delta r}{r}$  and  $\frac{\Delta l}{l}$ .

#### 6. Magneto-Resistance for Alternating Currents.

Lenard <sup>(324)</sup>, 1890, discovered that the resistance of *bismuth* in transverse fields from 6000 to 16,000 gauss increased more rapidly for direct currents than for alternating.

In *antimony* and *tellurium* in transverse fields, the resistance was greater for direct currents. In no case could a difference in resistance be detected for frequencies below 10,000 per sec.

Lenard failed to find any appreciable difference in the magneto-resistance of *iron*, *copper* and *German silver* for direct and alternating currents.

Zahn <sup>(608)</sup>, 1891, observed that the resistance of a *bismuth* spiral in a transverse field of 5000 gauss was about 0.6 % greater for alternating currents.

Sadowsky <sup>(466)</sup>, 1894, found that in strong fields, even with a frequency of 3 to 4 per sec., there existed a difference in the magneto-resistance of *bismuth* for direct and alternating currents.

Griffiths <sup>(221)</sup>, 1895, working with a frequency of 30 to 40 per sec., also found that the magneto-resistance of *bismuth* was greater for the alternating current in a transverse field. He attributed this increase in resistance to the fact that the ultimate particles or crystals of bismuth became magnetized in the field, that this magnetization increased with the field, and that the alternating current set up forced oscillations of the magnetized particles, which hindered the current.

Wachsmuth and Bamberger <sup>(558)</sup>, 1899, observed that, while the resistance of *bismuth* outside the field increased as the frequency rose from 29 to 110, in fields up to 10,000

gauss, the resistance was independent of the frequency. The resistance of antimony in a field of 8,250 gauss was found to be 1.5 % greater for alternating currents.

Eichhorn <sup>(462)</sup>, 1899, while measuring the resistance of a *bismuth spiral* in a varying magnetic field, observed a lag, or hysteresis, in the resistance, that depended on the rapidity of the variation of the field. This fact may vitiate the reliability of the bismuth spiral as a means for measuring the strength of changing magnetic fields.

In weak fields the resistance of *bismuth* was found by Bamberger <sup>(47)</sup>, 1901, to be less for alternating currents. When the field reached 4200 gauss, the resistance was the same for the two types of current. In the case of *antimony* the resistance for alternating currents was always smaller, both in and out of the field. For *iron* and *nickel* the resistances for the two currents were the same.

Simpson <sup>(483)</sup>, 1901—2, investigated the resistance of *bismuth* for alternating currents of different frequencies, in fields of various strengths, at temperatures from  $-185^{\circ}$  to  $+100^{\circ}$  C. He considered the change in resistance to be due to an E. M. F. set up in the bismuth by the magnetic field. The E. M. F. was found to lag behind the current producing it by an angle  $\alpha$ , where  $\tan \alpha = 1.13 + 13.2/n$ , in which  $n$  is the frequency of the current.

The frequencies varied from 10 to 60 cycles per sec. The E. M. F. set up at ordinary temperatures could be expressed by the equation:

$$\frac{e}{E} = (35 + 3.9n) (H - 4500) 10^{-8},$$

in which  $E$  is the impressed E. M. F., and  $e$  is the secondary E. M. F. set up by the field  $H$ . When  $H = 4500$ ,  $e = 0$ . For temperatures from  $-180^{\circ}$  C. to  $+100^{\circ}$  C., in a field of 700 gauss and for  $n = 30$ , the ratio  $e/E$  rose rapidly to a sharp maximum at  $-70^{\circ}$  C., and then decreased rapidly to a small value at  $100^{\circ}$  C.

Carpini <sup>(112)</sup>, 1904, measured the change in resistance of *bismuth* in direct and alternating fields. This change could be represented by the equation

$$I^2 = \frac{\Delta r}{r} \left( \frac{\Delta r}{r} 24783 + 168028 \right),$$

where  $I$  is the current through the electro-magnet.

Curve I, Fig. 98, shows the variation of  $\frac{\Delta r}{r}$  in a field of 50 cycles per sec., while Curve II gives the variation in a direct field.

Geipel <sup>(212)</sup>, 1912, while investigating the behavior of *bismuth*, *antimony* and *tellurium* carrying alternating and direct currents in a mag-

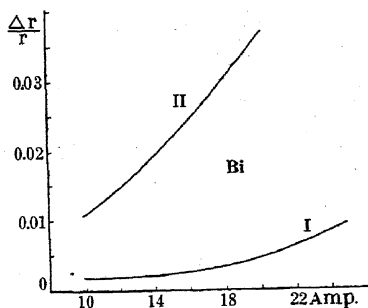


Fig. 98.

netic field, discovered that, when the current was turned off from the conductor, there existed a residual electromotive force, both without and within the magnetic field. The cause of the effect was supposed to be due to the existence of layers of material between the pure metal in the wire, and to some extent to the crystalline structure of the metal.

In the metals *nickel*, *copper*, *zinc*, *tin* and *mercury* no residual electromotive force was detected either in or out of the magnetic field.

To explain the difference in resistance of metals in a magnetic field, when measured with direct and alternating currents, Heurlinger <sup>(260)</sup>, 1916, developed the following equation connecting the coefficients of the Ettingshausen and Nernst effects,  $P$  and  $Q$ , with the resistance for alternating and direct currents in a magnetic field:

$$n \lim_{n \rightarrow \infty} (e_A \cdot C - e_D \cdot C) = -P \cdot Q H^2,$$

in which  $n$  is the frequency of the alternating current, and the other letters have their usual significance.



Heurlinger found that the above relation agreed with the observations of Zahn <sup>(609)</sup>, Seidler <sup>(475)</sup>, Geipel <sup>(211)</sup>, Lenard <sup>(324)</sup> and Bamberger <sup>(41)</sup>.

The change in resistance in *tellurium* in a transverse magnetic field was measured by Beckmann <sup>(55)</sup>, 1917. In fields of about 11,000 gauss and at 17° C., the constant A, in  $\frac{\Delta r}{r} = AH^2$ , varied from  $3.53 \times 10^{-11}$  to  $9.04 \times 10^{-11}$ .

In a field of 10,200 gauss, the change in resistance for A. C. and D. C. currents was:

$$\frac{\frac{\Delta r}{A \cdot C} - \frac{\Delta r}{D \cdot C}}{r_{D \cdot C}} = 7 \times 10^{-5}.$$

A month later the ratio was equal to  $3 \times 10^{-5}$ .

Wolff <sup>(605)</sup>, 1897; Sagnac <sup>(467)</sup>, 1902; Von Carolsfeld <sup>(111)</sup>, 1904; Gans <sup>(208)</sup>, 1906; König <sup>(295)</sup>, 1908; and Seidler <sup>(475)</sup>, 1910, have also studied the magneto-resistance for A. C. currents.

## 7. Magneto-Resistance in Liquid Metals.

Drude and Nernst <sup>(154)</sup>, 1891, observed that the resistance of *mercury* in a field of 8000 gauss increased about 0.2 %, while that of *molten bismuth* at 290° C. increased 0.4 %. The same year Des Coudres <sup>(132)</sup> called attention to the fact that this apparent increase might be attributed to heating of the metal by the current.

The work of Berndt <sup>(60)</sup>, 1907, seemed to confirm the contention of Des Coudres. He found that the smaller the capillary tube used, the less the increase in resistance. In fields from 1000 to 3000 gauss, the increase in resistance was not more than 0.00005 % for *mercury*, and 0.004 % for *molten bismuth*.

Rossi <sup>(461)</sup>, 1911, measured the resistance of *mercury*, and of *mercury-bismuth amalgams* in tubes of from 0.5 mm. to 0.7 mm. in diameter, in fields from 3350 to 4450 gauss. In both mercury and the amalgams the resistance was found

to increase with the field, the diameter of the tube and the concentration of the amalgam.

The above subject would seem to deserve further investigation.

#### 8. Magneto-Resistance in Electrolytes.

Neesen <sup>(386)</sup>, 1884, observed that weak magnetic fields caused a diminution in the resistance of a solution of *iron sulphate* placed parallel to the lines of magnetic force. In a transverse field no change was observed.

Lussana <sup>(348)</sup>, 1893, Hurmuzescu <sup>(292)</sup>, 1897, and Milani <sup>(307)</sup>, 1897, were unable to detect any influence of a magnetic field on the resistance of *salts of iron*.

Bagard <sup>(39)</sup>, 1899, observed an apparent increase of 1 % in the resistance of a *copper sulphate* solution in a transverse field of 5000 gauss.

Berndt <sup>(60)</sup>, 1907, examined a series of solutions of *salts of iron, nickel, cobalt, bismuth and copper*. No effect of the magnetic field on the resistance of those substances was detected in fields from 1000 to 3000 gauss.

Reed <sup>(427)</sup>, 1909, reported that after an accident to the machinery, it was found that on a submerged magnetic separator a peculiar corrosion had taken place on a part of the *copper tube* that surrounded the magnet. The portion of the tube corroded showed that the electrolytic current had flowed along the lines of the most intense magnetic flux, and that the electrolytic conductivity along these lines had been increased by the magnetic field.

Stifler <sup>(509)</sup>, 1909, examined the change in resistance of *cobaltous chloride, ferric chloride, and copper sulphate* solutions in a transverse magnetic field of 20,000 gauss. In the two chloride solutions there was an apparent decrease in resistance in the field, and this change was independent of the direction of the field and of the current. In the case of the copper solution the change in resistance was independent of the

direction of the field, but changed sign when the current through the electrolyte was reversed. This latter behavior led Stifler to attribute the apparent change in resistance in all three solutions to some secondary cause other than the magnetic field.

Svedberg <sup>(513)</sup>, 1914, investigated the change in resistance of *axoxyanisol*, *axoxyanisolphenetol*, and *axoxyphenetol* in a magnetic field. The conductivity was found to increase as the length of the molecule increased. As the "clearing point" was approached, the effect diminished, and at that point became zero. The effect required time to reach its maximum. In a field of 200 gauss 160 secs. were required, while in a field of 300 gauss 90 secs. were sufficient.

In general the conductivity was found to increase rapidly up to a field of 500 gauss, then less rapidly, and to approach a constant value in a field of 2000 gauss.

#### 9. Magneto-Resistance in Gases.

Boltzmann <sup>(85)</sup>, 1887, noticed that a magnetic field hindered the discharge in a *Geissler tube*, and that the current was reduced, as if the resistance had been increased ten fold.

Paalzow and Neesen <sup>(402)</sup>, 1897, also observed that the discharge in a *vacuum tube* was reduced by a transverse field, and sometimes extinguished.

Warburg <sup>(570)</sup>, 1897, found that a transverse field would sometimes delay the discharge in a *vacuum tube* several minutes.

According to Precht <sup>(421)</sup>, 1898, a spark discharge in *open air* between a point anode and a blunt cathode, under a potential of 8000 volts, had its potential reduced 1000 volts by a transverse field of 7000 gauss. If the cathode was pointed and the anode blunt, the potential in the same field was increased.

Birkeland <sup>(68)</sup>, 1898, found that at a pressure below 0.012 mm. a longitudinal field diminished greatly the potential

difference necessary to cause a *discharge in a tube*. When the magnetic field at the cathode reached a critical value, the sparking potential fell to 1/10 its original value.

Willows <sup>(591)</sup>, 1901—5, called attention to the fact that, for pressures above a critical value, a transverse field reduced the current through the *tube*; while for pressures below the critical, the magnetic field near the cathode reduced the potential difference and increased the current.

Almy <sup>(11)</sup>, 1901, discovered that a longitudinal as well as a transverse field reduced the necessary sparking potential in a *discharge tube*.

Stark <sup>(503)</sup>, 1903, noticed that both transverse and longitudinal fields reduced the resistance in the case of *glow discharges*. Pellat <sup>(409-13)</sup>, 1902—4, found that both transverse and longitudinal fields when applied either at the anode or cathode region increased the resistance, the transverse field producing the larger increase.

Peck <sup>(407)</sup>, 1905, in his study of the influence of a magnetic field on the discharge in a *vacuum tube*, employed a cathode covered with calcium oxide. When the cathode fall in potential was large, the resistance was decreased by the field; but when the fall was small the resistance was increased near the cathode, as at other portions of the tube.

Righi <sup>(450)</sup>, 1911—12, observed some rather unusual changes in potential in a *discharge tube* with increase in the magnetic field. In fields from 600 to 1100 gauss, the potential difference increased until the potential was 1900 volts, then the potential suddenly dropped to 1700 volts. The potential then continued to increase again as the field rose to 2500.

In the interval between 1900 and 1700 volts, Righi's so-called "*magnetic rays*" were supposed to be formed by what he termed magneto-ionization. The rays thus formed were composed of atom-electron doublets accompanied by ions and electrons.

More and Mauchly <sup>(369)</sup>, 1913, claimed that all the pheno-

mena observed by Righi in connection with the "*magnetic rays*" can be produced without a magnetic field, if the hot Wehnelt cathode and a wire gauze about the tube are connected to earth. They thought that the magnetic field did not change the nature of the discharge, but simply displaced it from the cathode.

Stassano <sup>(504)</sup>, 1912, established the following facts relative to the *conductivity of gases* in a magnetic field.

1. Very weak fields facilitate the electric discharge in tubes of rarefied air at the critical pressure for maximum electrical conductivity.

2. Weak fields increase the luminescence in the tube. Stronger fields reduce the luminescence, and, when high enough, extinguish it entirely.

3. When the vacuity is increased, the effect of feeble fields on the discharge is rendered void. For vacua of the order of 0.001 mm., the discharge is aided by fields as high as 100 gauss; while for vacua of the order of 0.01 mm., more intense fields are required to affect the discharge.

Thus it is seen that a magnetic field aids the discharge in rarefied gases, so long as a certain value of the field strength is not exceeded, and that after this critical value is passed, the magnetic field hinders the discharge. The above facts may explain why some investigators have found that the magnetic field assisted the discharge in vacuum tubes, while others have found that the field hindered the discharge. They probably worked under different conditions of pressure and field.

Lehmann <sup>(323)</sup>, Bloch <sup>(71)</sup>, and Gouy <sup>(219)</sup> had previously noted the opposite effect of weak and strong magnetic fields on the discharge in *vacuum tubes*, but Gouy was the only one who had made reference to the influence of the degree of the vacuum on the discharge in the magnetic field. The size and shape of the tube also exert an influence on the discharge.

Kent and Frye <sup>(284)</sup>, 1913, while studying the changes that take place in the spectrum from a *capillary discharge tube* in a magnetic field, found that the resistance of the tube was increased.

Earhart <sup>(157-8)</sup>, 1914, studied the electric discharge, in a longitudinal field, between parallel plates of a condenser, in atmospheres of *hydrogen*, *air*, and *carbon dioxide*, at pressures above and below the critical pressure. He found that in general, weak fields increased the current, while strong fields reduced the current. It required a greater critical magnetic field to secure a decrease in the current for lower gas pressures. Near the critical pressure the magnetic field had little effect on the current. For pressures above the critical, the longitudinal field reduced the current.

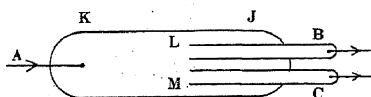


Fig. 99.

In his study of the deviation of electrons and ions in a magnetic field, Righi <sup>(452)</sup>, 1915, used a bifurcated tube as shown in Fig. 99.

The portion *KJ* was 14 cm. long and 5.3 cm. in diameter. The branch tubes *L* and *M* were 14 cm. long and 1.5 cm in diameter. The pressure of air in the tube varied from 0.02 mm. to 3 mm. The current through the tube varied from 1 to 5 milliamperes. The electrodes *B* and *C* were connected to a differential galvanometer of small resistance.

In a magnetic field the current in the tube was deviated, and a deflection of the galvanometer resulted.

Ives <sup>(273)</sup>, 1917, found that he could reproduce the "*magnetic rays*" of Righi without the use of a magnetic field. With a spark gap in the electric circuit the so-called rays could be produced. If the pressure were low enough in the tube, the rays could be produced without either magnetic field or spark gap.

The magnetic field only served to interrupt the discharge in the tube, and after each interruption, the tube contained an initial positive charge and a residual negative charge.

The general effect of a transverse magnetic field on a discharge tube Ives <sup>(274)</sup>, 1918, found to be, that when the field acted on the cathode region only, the current through the tube was increased; when the field acted on the anode region only, the current was decreased; when the whole tube was in the field, the current increased in value.

#### 10. Magneto-Resistance in Flames.

Wilson <sup>(595)</sup>, 1909, investigated the influence of a magnetic field on the conductivity of a *Bunsen flame*. The horizontal electric current and the vertical flow of gases were perpendicular to the magnetic lines of force. The ratio of the potential difference between the exploring electrodes to the current was taken as the resistance of the flame. The curve

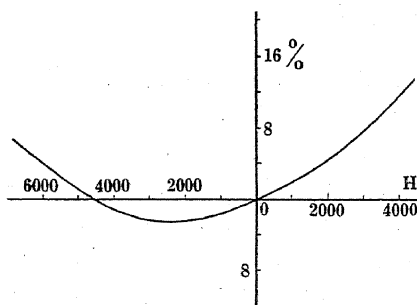


Fig. 100.

in Fig. 100 gives the percentage change in resistance with the magnetic field. The lack of symmetry is due to the upward movement of the flame. The following equation was found to give the relation between the change in resistance and the field strength:

$$100 \frac{\Delta r}{r} = 3.1 \times 10^{-7} H^2 + 1.5 \times 10^{-8} H.$$

The expression for the conductivity was:  $\frac{\Delta \sigma}{\sigma} = \frac{\Delta K}{K} - \frac{HV}{X}$ , in which  $\sigma$  is the conductivity,  $K$  the velocity of the negative ions, under unit electric field,  $H$  the field,  $V$  the upward velocity of the gas in the flame, and  $X$  the difference in potential between the horizontal electrodes.

To avoid the complications Wilson encountered, due to the upward movement of the gases in the flame, Heaps<sup>(250)</sup>, 1916, caused the electric current to flow vertically upward in the flame, instead of horizontally. The apparatus employed is shown in Fig. 101. The flame  $F$  was produced by burning a mixture of gasoline vapor and air in a Bunsen burner. The electric current was supplied from a battery,  $B$ , capable of giving about 1000 volts. The cathode,  $K$ , and the anode,  $A$ , were of platinum. The cathode was

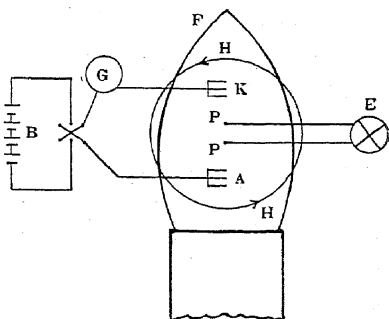


Fig. 101.

covered with metallic oxides from burnt sealing wax, to prevent the large cathode fall in potential. The potential gradient was measured by means of two platinum wires,  $P, P$ , connected to an electrometer,  $E$ . The circle  $HH$  represents the magnetic field. The variation of the current in the flame, when the magnetic field was on and off, is shown by the curves in Fig. 102. The curves are plotted in terms of galvanometer deflections against the applied voltage.

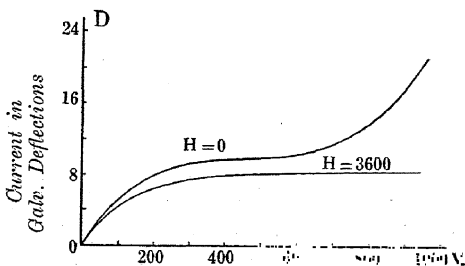


Fig. 102.

The general effect of the magnetic field is to reduce the current and increase the resistance. The current soon reaches a constant value. The apparent saturation is due to the fact that an anode fall of potential appears, and ionization by collision occurs for the higher voltages.



## 11. Theories of Change of Resistance in a Magnetic Field.

Several theories to account for the change in resistance of conductors in a magnetic field have been proposed and equations have been deduced to represent this change.

The leading theories are here briefly given.

Sir J. J. Thomson <sup>(520)</sup>, 1909, in his theory for the change in resistance of a conductor in a magnetic field, assumed that the electrons behave as a perfect gas, that they have a mean free path and velocity, and that they exert a pressure. Under a transverse magnetic field the electrons move in cycloids, hence their paths and consequently the resistance of the conductor are increased. In a longitudinal field, the electrons describe spirals, and again the paths and the resistance are increased.

Thomson deduced the following expression for the change in resistance in a magnetic field:

$$\frac{\Delta r}{r} = \frac{1}{2} H^2 u^2,$$

where  $u$  is the velocity of an electron under a unit electric field.

$$\text{Since } u = \frac{1}{2} \frac{e}{m} T,$$

$$\frac{\Delta r}{r} = \frac{1}{12} \left( \frac{e}{m} \right)^2 H^2 T^2,$$

in with  $T$  is the mean period of the electrons.

The change in resistance accordingly varies as the square of the field and the square of the mean free period of the electron, and is an increase.

Van Everdingen <sup>(187)</sup>, 1901, like Thomson, supposed the current to be carried by electrons that behaved like a perfect gas, but also assumed that the collisions between electrons and molecules are similar to the collisions between hard elastic bodies, and that the electrons between two collisions are free from the action of all forces except the electric force that drives the current through the conductor. His theory

calls for a decrease in resistance in a transverse magnetic field. For this decrease he obtained the following expression:

$$\frac{\Delta r}{r} = -\frac{63}{16} H^2 u^2.$$

Thomson <sup>(522)</sup>, 1902, pointed out that Van Everdingen's suppositions in regard to the behavior of electrons in a conductor were probably at fault, that the collisions between electrons and molecules are not similar to collisions between elastic solids, and that the electrons, being within a distance of  $10^{-7}$  cm. of the molecules around them, are acted upon by molecular forces far greater than that of the external electric field. Hence the electron rushes into or past the molecule with a velocity much greater than that with which it started on its journey.

In accordance with the above statement, Thomson deduced the following expression for the effect of the magnetic field on the current flowing in the conductor under an electric force  $X$ :

$$-\frac{1}{24} \left( \frac{e}{m} \right)^3 H^2 \Sigma XT^3,$$

in which  $\Sigma_n$  is the number of electrons. Since the current is decreased by the field, the resistance is increased.

Gans <sup>(208)</sup>, 1906, deduced the following expression, based on the electron theory, for the isothermal change in resistance in a magnetic field:

$$\frac{\Delta r}{r} = \frac{9\pi}{64} (4 - \pi) e^2 v^2 H^2,$$

in which  $v$  is the velocity of an electron under a unit force.

For the adiabatic change in resistance this equation was obtained:

$$\frac{\Delta r}{r} = \frac{9\pi}{16} \left( \frac{32 - 9\pi}{32} \right) e^2 v^2 H^2.$$

The change in resistance in each case is an increase.

Adams <sup>(4)</sup>, 1907, starting with the assumption that the magnetic field brought about a rearrangement of the molecules,

such that the mean free path and the period of the electron were altered, modified Thomson's theory for the change in resistance, and from Thomson's equations deduced this expression for the change of resistance in a magnetic field:

$$\frac{\Delta r}{r} = \frac{\Delta T}{T} - \frac{1}{4} H^2 \left( \frac{e}{m} \right)^2 T^2.$$

Here  $\Delta T = T - T_H$ , in which  $T$  is the mean free path period of an electron in zero field, and  $T_H$  the period in the field  $H$ . If  $\frac{\Delta T}{T}$  is positive and larger than the second term, the change in resistance will be an increase, otherwise a decrease. The fact that  $\Delta T$  must be positive, to bring about an increase in resistance, means that the magnetic field sets up such a new molecular arrangement that the time between the collisions of electrons and molecules is reduced. In the paramagnetic (except iron, nickel, and cobalt) and in the diamagnetic substances, the susceptibility is constant and independent of the field strength. Hence the change in molecular arrangement would increase with the field, and likewise the resistance would increase for all fields. This is in accord with observation.

In the case of the ferromagnetic metals, however, the metals undergo magnetic saturation. All the way up to saturation, the molecular arrangement is being continually changed, and finally reaches a steady state, when the  $\frac{\Delta T}{T}$  term acquires its greatest value. Then as  $H$  is further increased, the second term of the above equation increases until it is equal to the first, and then becomes greater. Consequently the change in resistance, which at first increases, becomes zero, and then decreases. This change in sign of  $\frac{\Delta r}{r}$  in a transverse field, in the case of the ferromagnetic metals, has been attributed by Jones and Malam<sup>(280)</sup>, p. 186, to the fact that the metal was not exactly perpendicular to the magnetic lines of force.

Richardson <sup>(433)</sup>, 1914, employing the same assumption as that used by Adams, deduced the following equation for the change in resistance in a magnetic field:

$$\frac{\Delta r}{r} = -\frac{1}{T_0} \left( \Delta T - \frac{1}{12} H^2 \frac{e^2}{m^2} T^3 \right),$$

in which  $T_0$  is the free period of the electron outside the magnetic field, and  $T$  the period in the field. Since  $T_0$  and  $T$  are nearly equal, we may write

$$\frac{\Delta r}{r} = -\frac{\Delta T}{T} + \frac{1}{12} H^2 \frac{e^2}{m^2} T^2,$$

an expression similar to that of Adams. The above equation may be written

$$\frac{\Delta r}{r} = -\frac{\Delta T}{T} + \frac{1}{3} \frac{H^2}{C^2} \left( \frac{\sigma}{n e} \right)^2,$$

in which  $\sigma$  is the electrical conductivity. If  $\frac{\Delta T}{T}$  is neglected, the second term affords a means of determining  $n$ , the number of electrons in a unit volume.

Livens <sup>(329)</sup>, 1915, starting with the usual suppositions with regard to free electrons, deduced for the change in resistance in a transverse magnetic field the expression:

$$\frac{\Delta r}{r} = \left( 1 - \frac{\pi l}{4} \right) \frac{e^2 l^2 q}{m^2 c^2},$$

in which  $l$  is the mean free path of the electron when undisturbed, and  $q = \frac{3}{2} u^2$ , where  $u^2$  is the mean square velocity of the electron. According to the above, the change in resistance in a transverse field is an increase.

For the change in resistance in a longitudinal field, Heap's <sup>(251)</sup>, 1917, obtained the equation:

$$\frac{\delta r}{r} = -\frac{\delta \lambda}{\lambda_0},$$

in which  $\lambda_0$  is the mean free path of the electron in zero field, and  $\delta \lambda$  is the change in path in the field. If the longitudinal field produces a decrease in the mean free path, an increase in resistance will result. Practically all observations thus far made have indicated an increase in resistance in a longitudinal field.

For the change in resistance in a transverse field Heaps deduced this equation:

$$\frac{\Delta r}{r} = H^2 \left( \frac{e}{m} \right)^2 \frac{\lambda_0^2}{v^2} - \frac{d\lambda}{\lambda_0},$$

in which  $v$  is the velocity of agitation of the electrons. According to the above equation, a transverse field will produce an increase in resistance, so long as  $\frac{d\lambda}{\lambda_0}$ , if positive, is not greater than  $H^2 \left( \frac{e}{m} \right)^2 \frac{\lambda_0^2}{v^2}$ .

The difference between the two effects produced by the two fields is:

$$\frac{\Delta r}{r} - \frac{\delta r}{r} = H^2 \left( \frac{e}{m} \right)^2 \frac{\lambda_0^2}{v^2} - \frac{d\lambda}{\lambda_0} + \frac{\delta\lambda}{\lambda_0}.$$

For crystalline substances, or those drawn through plates, the two last terms are not equal, but for substances that are magnetically isotropic, they may be considered equal, and in that case,

$$\frac{\Delta r}{r} - \frac{\delta r}{r} = H^2 \left( \frac{e}{m} \right)^2 \frac{\lambda_0^2}{v^2}, \text{ or}$$

$$\frac{\Delta r}{r} - \frac{\delta r}{r} = H^2 \left( \frac{e}{m} \right)^2 T^2.$$

According to this equation, the transverse field produces a greater increase in resistance than a longitudinal. All experiments seem to confirm the above relation, except in the case of the ferromagnetic metals and bismuth. It is conceivable that some exception may be found in other highly crystalline substances.

Patterson<sup>(405)</sup>, Laws<sup>(311)</sup>, Heaps<sup>(251)</sup>, and others have made use of the several  $\frac{\Delta r}{r}$  equations, already referred to, in computing the mean free path  $\lambda_0$ , the mean free period  $T$ , and the velocity  $u$  of electrons, and  $n$  the number of electrons in a unit volume. The results obtained are of the same order of magnitude as those obtained by other means.

The following expression, based upon the electron theory,

for the conductivity of a metal in a magnetic field, was deduced by La Rosa <sup>(457)</sup>, 1919:

$$\sigma = \frac{e^2 v}{4 \alpha T} l_0 n_H \left[ 1 + \frac{1}{6} \left( \frac{e l_0 H}{2 m v} \right)^2 \right],$$

in which  $e$ ,  $\alpha$ ,  $v$ ,  $m$ ,  $H$ ,  $T$ , have the usual significance, and  $l_0$  is the mean free path of the electrons in zero field, and  $n_H$  the number of electrons in  $H$  field. By means of this equation La Rosa computed the mean free path of the electrons in bismuth in zero field to be  $l_0 > 4 \times 10^{-6}$  cm.

## 12. Magneto-Resistance and the Hall Effect.

The change in resistance in a conductor in a magnetic field may be considered as a difference of potential set up by the field, and hence is sometimes termed the "longitudinal Hall effect". Attention has already been called to the striking analogy between the Hall effect and the change of resistance in a magnetic field.

For *pure bismuth*, the following relation between the Hall electromotive force and the change in resistance in a magnetic field was found by Beattie <sup>(49)</sup>, 1896, to hold good:

$$E = C \sqrt{\Delta r},$$

in which  $C$  is a constant.

For *impure specimens of bismuth*, in which the Hall effect reached a maximum, the following equation was found to hold:

$$\pm E = C_1 (\Delta r)^{1/2} + C_2 (\Delta r)^{3/2},$$

in which  $C_1$  and  $C_2$  are constants.

This latter equation permits a change in sign of the Hall effect.

From the equations of Drude <sup>(149)</sup>, it may be shown that the change in resistance in a magnetic field and the Hall effect are related by this equation:

$$\frac{\Delta r}{r} = \left( \frac{R}{r} \right)^2,$$

in which  $R$  is the Hall coefficient, and  $r$  the specific resistance of the metal.

Gans <sup>(208)</sup>, 1906, deduced equations similar to the above:

For the isothermal Hall effect:

$$\frac{\Delta r}{r} = 0.27 \left( \frac{R_i}{r} \right)^2.$$

For the adiabatic Hall effect:

$$\frac{\Delta r}{r} = 0.15 \left( \frac{R_a}{r} \right)^2.$$

On the basis of the existence of two kinds of ions, Corbino <sup>(130)</sup>, 1918, developed the following relation between the Hall effect,  $R$ , and the change in electrical conductivity,  $C$ , in a magnetic field:

$$R \cdot C = R_0 \cdot C_0 - e(v_2 - v_1) \frac{\Delta C}{C_0},$$

in which  $R_0$  and  $C_0$  are the Hall coefficient and the electrical conductivity in zero field, and  $v_1$  and  $v_2$  the mobilities of the positive and negative ions.

Corbino also deduced this expression for the change in conductivity in a magnetic field:

$$\frac{\Delta C}{C_0} = -b_1 b_2 (v_1 + v_2)^2 H^2, \text{ in which}$$

$$b_1 = \frac{N_1 v_1}{N_1 v_1 + N_2 v_2}, \quad b_2 = \frac{N_2 v_2}{N_1 v_1 + N_2 v_2}.$$

By means of these equations Trabacchi <sup>(546)</sup>, 1918, determined for *bismuth* the number of ions,  $N_1$  and  $N_2$ , per 1 c.c., and their mobilities at  $16^\circ$  and  $-65^\circ$  C. The table below contains the results.

Temperature	$N_1$	$N_2$	$v_1$	$v_2$
$+16^\circ$ C.	$0.5 \times 10^{18}$	$1.8 \times 10^{18}$	$5.4 \times 10^{15}$	$15.6 \times 10^{15}$
$-65^\circ$ „	0.24 „	0.98 „	13.9 „	41.6 „

The change in the Hall effect and in the resistance of the same *bismuth* plate in a magnetic field was also determined by Trabacchi as follows:

$H$	$R$	$r_H/r$
5770	-5.45	1.17
9140	-4.67	1.34
13680	-4.15	1.55

# CHAPTER XI

## 1. Galvanomagnetic Longitudinal Temperature Difference.

Nernst <sup>(387)</sup>, 1887, discovered that a *bismuth* plate, carrying an electric current, when placed in a magnetic field of about 8000 gauss, exhibited a longitudinal temperature gradient.

If an electric current,  $I$ , is flowing through a plate,  $P$ , Fig. 103, and this plate is subjected to a magnetic field,  $H$ , a temperature difference,  $\Delta T$ , between the points  $A$  and  $B$  will be set up, which may be written

$$\Delta T = L \frac{lHI}{wt},$$

in which  $l$  is the distance between  $A$  and  $B$ ,  $w$  the width and  $t$  the thickness of the plate, and  $L$  a constant depending on the metal.

In bismuth plate No. I, examined by Nernst, the temperature gradient was opposed to the electric current  $I$ , while in plate No. III the gradient was in the same direction as the current.

The longitudinal thermomagnetic E.M.F. in plate I was found to be positive (p. 244), while that in plate III was negative. So that the E.M.F. set up in each plate, in the presence of the temperature gradient, by the magnetic field, opposed the flow of the electric current,  $I$ .

Ettingshausen <sup>(169)</sup>, 1888, also observed a longitudinal temperature difference in *bismuth* in a field of 9000 gauss.

Zahn <sup>(609)</sup>, 1904, found that the longitudinal temperature difference in *bismuth* was proportional to the primary electric current, and that it increased with the field strength; but the exact variation with the field was not determined. In an impure piece of bismuth the temperature gradient was opposed to the direction of the electric current, while in a pure specimen the gradient was in the same direction as the electric current. With a current of 1 ampere through the plate. a field of 10,000 gauss set up a temperature gradient of 0.04 °C,

Hall and Campbell <sup>(247)</sup>, 1911, sought for a longitudinal

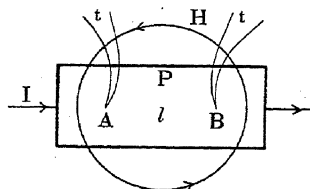


Fig. 103.



temperature difference in soft *iron*. The effect, if present, was too small to be measured with certainty.

So little work has been done on this longitudinal galvanomagnetic temperature difference, that no general conclusions can be drawn as to its direction or variation with field strength.

According to the equation

$$\Delta T = L \frac{lHI}{wt},$$

the difference in temperature is more likely to be detected when  $l$ ,  $I$ , and  $H$  are large, and  $w$  and  $t$  are small.

## 2. Theory of the Galvanomagnetic Longitudinal Temperature Difference.

Drude <sup>(149)</sup>, 1900, pointed out that since the longitudinal galvanomagnetic and thermomagnetic effects are not reversed in sign on the reversal of the magnetic field, those effects should vary as the square of the field strength.

With the usual assumptions of the dual electron theory, Drude deduced the following equations, which include the longitudinal change in temperature,  $\frac{\delta T}{\delta x}$  along the plate, in the direction of the electric current:

$$\begin{aligned}\frac{d\xi_1}{dt} &= v_1 \left( e_1 X - \frac{4\alpha T}{3} \cdot \frac{d \log N_1}{dT} \cdot \frac{\delta T}{\delta x} \right), \\ \frac{d\xi_2}{dt} &= v_2 \left( e_2 X - \frac{4\alpha T}{3} \cdot \frac{d \log N_2}{dT} \cdot \frac{\delta T}{\delta x} \right),\end{aligned}$$

$$I = e_1 N_1 \frac{d\xi_1}{dt} + e_2 N_2 \frac{d\xi_2}{dt},$$

in which  $\frac{d\xi_1}{dt}$  and  $\frac{d\xi_2}{dt}$  are the current velocities of the two types of electrons, and  $v_1$  and  $v_2$  their velocities under unit force, and  $X$  the electric force along the plate. The other letters have their usual significance.

If the portion of the metal in the magnetic field is considered as a different metal from that outside the field (and the properties of the same metal within and without the field are different), then the galvanomagnetic longitudinal temperature difference is in reality a Peltier effect.

## B. THERMOMAGNETIC PHENOMENA

### CHAPTER XII

#### NERNST EFFECT: TRANSVERSE THERMOMAGNETIC ELECTROMOTIVE FORCE

##### 1. Historical and General.

While studying the Hall effect in *bismuth*, Ettingshausen and Nernst <sup>(172)</sup>, 1886, were led by certain irregularities in the effect to examine the influence of the magnetic field on a bismuth plate through which a heat current was flowing. They used a rectangular plate of bismuth about 5 cm. long, 4 cm. wide, and 0.2 cm. thick. The plate was held by two copper clamps, *C, C*, Fig. 104,

one of which was heated and the other was kept cold. Copper wire electrodes led from the edges of the plate, at *A* and *B*, to a galvanometer. The bismuth plate was placed with its plane perpendicular to the

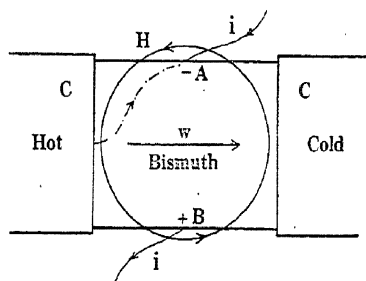


Fig. 104.

magnetic field, *H*. When the magnetic field was put on, an electric difference of potential was set up between the points *B* and *A*, and an electric current, *i*, continued to flow from *B* through the galvanometer to *A* as long as the field was on. The direction of this current changed with the direction of the field, and also with the direction of the heat current in the plate. If the two ends of the plate were equally heated, the effect disappeared.

The electromotive force set up by the magnetic field was found to be proportional to the magnetic field strength, to the distance between the side electrodes, to the temperature gradient along the plate; and independent of the thickness of the plate.

This transverse thermomagnetic electromotive force may be expressed by the following relation:

$$E = Q \cdot H \cdot b \frac{dt}{dl},$$

in which  $Q$  is the Nernst constant depending on the metal employed,  $H$  the magnetic field strength,  $b$  the width of the plate, and  $\frac{dt}{dl}$  the temperature gradient along the plate.

The above equation may be written

$$E = \frac{Q \cdot A \cdot H}{K \cdot d},$$

where  $Q$  and  $H$  are as before,  $K$  is the thermal conductivity,  $A = K \cdot a \cdot \frac{dt}{dl}$ ,  $a = bd$ , and  $d$  = the thickness of the plate.

Or we may write

$$Q = \frac{E \cdot l}{H \cdot b (t_2 - t_1)},$$

in which  $l$  is the distance between two points along the plate at temperatures  $t_2$  and  $t_1$ .

If  $W$  is the amount of heat that passes through a cross-section of the plate per sec., then

$$W = \frac{K(t_2 - t_1) b \cdot d}{l}.$$

Hence

$$\frac{Q}{K} = \frac{E \cdot d}{W \cdot H}.$$

The Hall coefficient is

$$R = \frac{E \cdot d}{I \cdot H}.$$

Here the analogy between  $\frac{Q}{K}$  and  $R$  is complete. The heat current  $W$  in the one corresponding to the electric current  $I$  in the other.

In the following table of Nernst and Hall effects, determined by Ettingshausen and by Nernst <sup>(387)</sup>, 1887, it will be found that  $\frac{Q}{K}$  and  $R$  agree with each other better than  $Q$  and  $R$ . The Nernst effect was determined while the ends of the plate were at about  $100^{\circ}$  and  $12^{\circ}$  C., or at a mean temperature at the middle of the plate of about  $56^{\circ}$  C. The Hall effect was probably determined at room temperature.

*Nernst and Hall Effects Compared*

Substance	$Q$	Temperature Coefficient	$\frac{Q}{K}$	$R$
Bismuth . . .	-0.132	-0.0133	-7.8	-10.1
Antimony . .	-0.00887	+0.00163	-0.21	+0.192
Nickel . . . .	-0.00861	+0.00402	-0.066	-0.024
Cobalt . . . .	-0.00224	+0.00839	-0.013	+0.0046
Iron . . . . .	+0.00156	—	+0.0096	+0.0113
Steel . . . . .	+0.000706	+0.00400	+0.0071	+0.0175
Copper . . . .	+0.000090	+0.0050	+0.00013	+0.00052
Zinc . . . . .	+0.000054	—	+0.00020	-0.00041
Silver . . . . .	+0.000046	—	+0.00005	+0.00083
Lead . . . . .	(?) 0.000005	—	+0.00006	+0.00009
Tin . . . . .	(?) +0.000004	—	+0.00003	-0.00004
Carbon . . . .	(-0.0001)	—	(-0.25)	-0.18

The lack of better agreement between  $\frac{Q}{K}$  and  $R$  may be due in part to the fact that  $Q$  has not been corrected for the Righi-Leduc effect, p. 233, and  $R$  for the Ettingshausen effect, p. 146. Furthermore, the thermal conductivity  $K$  probably was not determined for the particular piece of metal employed for finding  $Q$  and  $R$ .

## 2. The Sign of the Nernst Effect.

### *a. Nernst's Convention.*

According to the convention laid down by Nernst <sup>(387)</sup>, a metal is said to have a *positive* Nernst effect, when in going from the entrance of the heat current,  $W$ , Fig. 105, to the point of entrance of the derived electric current,  $i$ , at the

edge of the plate,  $B$ , the passage is in the direction of the electric current in the electromagnet  $H$ . In accordance with this convention, the Nernst effect in *iron* is positive.

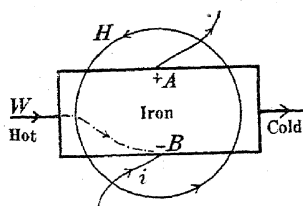


Fig. 105.

In the case of *bismuth*, the passage from the entrance of the heat current,  $W$ , Fig. 104, to the point of entrance of the derived electric current,  $i$ , at  $A$ , is in a direction opposed to the current in the electromagnet, and accordingly the Nernst effect in this metal is called *negative*.

#### b. Later Convention.

The more recent convention for the sign of the Nernst effect is the reverse of that originally adopted by Nernst, and is analogous to the convention for the sign of the Hall effect, p. 9. That is, if the passage from the point of entrance of the heat current,  $W$ , Fig. 106, to the point of higher potential at the edge of the plate,  $B$ , is in the direction of the current in the electromagnet,  $H$ , the effect is said to be *positive*, as in *bismuth*.

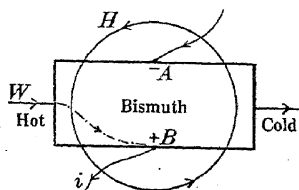


Fig. 106.

In accordance with this convention, the passage from the point of entrance of the heat current to the point of higher potential,  $A$ , in *iron*, Fig. 105, is opposed to the current in the electromagnet, and the effect is called *negative*.

This later convention has been adopted by Hall <sup>(247)</sup>, Richardson <sup>(433)</sup>, Smith <sup>(493)</sup>, and others, by the *Recueil des Constantes Physiques*, and by the *Smithsonian Physical Tables*, 6th Edition. In the tables of Nernst effects, p. 230, the signs are given in accordance with the later convention, while the quoted values in the text have the signs as given by the authors.

### 3. Variation of the Nernst Effect with the Magnetic Field and with Temperature.

*Bismuth.* Ettingshausen and Nernst <sup>(173)</sup>, 1887, observed that the Nernst effect in *bismuth* decreased slowly with increase of the magnetic field. In *bismuth-tin alloys* the effect decreased with increase of field more rapidly than in the pure metal.

Everdingen <sup>(182)</sup>, 1898, determined the variation of the Nernst effect with the magnetic field in *electrolytic bismuth*. The curve in Fig. 107 represents the change of the total Nernst E. M. F. with the field  $H$ . The electromotive force reaches a negative maximum, then decreases, changes sign, and seems to increase indefinitely. The temperature at the middle of the plate was about  $34^{\circ}\text{C}$ . The E. M. F. was in absolute units.

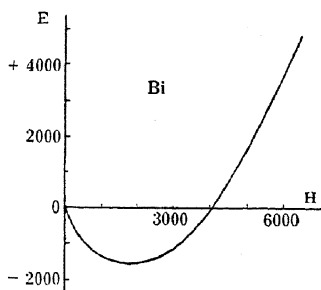


Fig. 107.

The change in the Nernst effect in *electrolytic bismuth* with increase in the magnetic field was also investigated by Yamaguchi <sup>(607)</sup>, 1900. The following tables gives the results. The values of  $m$  are computed from the equation,

$$E = -b \frac{dt}{dl} m,$$

where  $m = QH$ . The E. M. F. was measured in microvolts.

#### *Nernst Effect in Bismuth*

Temperature	$-119^{\circ}\text{C}$ .	$-30.5^{\circ}\text{C}$ .	$10^{\circ}\text{C}$ .	$62^{\circ}\text{C}$ .
$H$	$m$	$m$	$m$	$m$
1293	—	—	+ 0.46	— 0.30
2650	+ 25	+ 7	2.32	— 0.25
4120	38	14	5.50	+ 0.65
6120	54.5	23	10.60	+ 2.82
8650	72	31	15.1	—

According to the curves in Fig. 108, it is evident that  $m$  increases with the field more rapidly at the lower temperatures. At  $62^{\circ}\text{C.}$  the effect changes sign, as Everdingen found was the case at  $34^{\circ}\text{C.}$  and  $50^{\circ}\text{C.}$  Yamaguchi also determined the change in resistance in the magnetic field. The

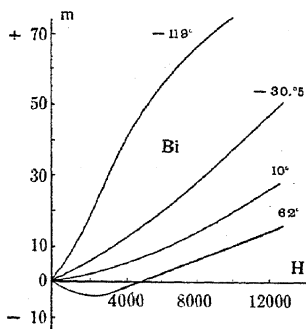


Fig. 108.

values for  $\frac{\Delta r}{r}$  and

$$m \div \frac{\Delta r}{r}$$

are given in the table below.

Nernst Effect and Change in Resistance in Bismuth

	$-119^{\circ}\text{C.}$		$-30.5^{\circ}\text{C.}$		$10^{\circ}\text{C.}$		$62^{\circ}\text{C.}$	
$H$	$\frac{\Delta r}{r}$	$m \div \frac{\Delta r}{r}$	$\frac{\Delta r}{r}$	$m \div \frac{\Delta r}{r}$	$\frac{\Delta r}{r}$	$m \div \frac{\Delta r}{r}$	$\frac{\Delta r}{r}$	$m \div \frac{\Delta r}{r}$
1293	—	—	—	—	0.047	9.8	0.010	-29
2650	0.90	28	0.22	32	0.123	19	0.040	-6
4120	1.39	27	0.39	35	0.170	32	0.096	+6.7
6120	2.21	24.5	0.64	35	0.337	31	0.103	+27
8650	3.04	24	0.92	34	0.494	31	—	—

The change in resistance is thus seen to increase with the field strength, while the ratio  $m \div \frac{\Delta r}{r}$  is fairly constant, except for weak fields, and for the case where the Nernst effect changes sign. It would seem then, that, as in the case of the Hall effect, the change in resistance of the metal in the magnetic field has a direct bearing on the Nernst effect.

Lownds<sup>(344)</sup>, 1901, determined the Nernst effect in a crystal of bismuth, both when the heat current was parallel to the crystallographic axis and when it was perpendicular to this axis. The determinations were made at temperatures from  $-140.5^{\circ}$  to  $+73^{\circ}\text{C.}$ , and in fields up to 7000 gauss. The effect was considerably larger when the heat flow was

perpendicular to the crystal axis, and in general the effect was greater at the lower temperatures. At  $73^{\circ}\text{C}$ . the effect underwent a negative maximum, followed by a change in sign, as Van Everdingen (p. 215) had found was the case at  $34^{\circ}\text{C}$ ., and Yamaguchi (p. 216) at  $62^{\circ}\text{C}$ .

*Antimony.* The variation of the Nernst effect in *antimony*, with field strength and with temperature, given in the table below, is due to Barlow <sup>(44)</sup>, 1903.

Nernst Effect in Antimony

$H$	$-75^{\circ}\text{C}$ .	$-19^{\circ}\text{C}$ .	$+39^{\circ}\text{C}$ .
	$m = QH$	$m = QH$	$m = QH$
4,310	—	+ 72	—
8,190	+ 174	+ 159	+ 113
10,200	+ 222	+ 202	+ 147
12,500	+ 281	+ 243	+ 185

Here  $m$  increases slightly more rapidly than  $H$ , and is greater at the lower temperatures.

Van Aubel <sup>(31)</sup>, 1911, measured the relative magnitudes of the Nernst effect in *antimony* and *graphite* at  $40^{\circ}\text{C}$ ., with the result that the effect in *graphite* was about 2.63 times that in *antimony*.

*Iron.* Hall and Campbell <sup>(247)</sup>, 1911, measured the Nernst effect in soft *iron* with the following results.

Nernst Effect in Soft Iron

$H$	Temperature	$Q$
5550	$31^{\circ}\text{C}$ .	— 0.00086
5500	$60^{\circ}$	— 0.00098

Here  $Q$  increases with temperature.

*Nickel and Cobalt.* The Nernst effects in *nickel* and *cobalt*, at temperatures from  $56^{\circ}$  to  $410^{\circ}\text{C}$ ., and from  $56^{\circ}$  to  $550^{\circ}\text{C}$ ., were determined by Smith <sup>(488)</sup>, 1911. The variation of the Nernst effect in *nickel* with temperature and field



strength is shown in Fig. 109. For any given temperature below the critical, the Nernst effect in *nickel* is seen to be at first proportional to the magnetic field. After the maximum magnetization of the metal has been reached, the effect approaches a saturation value.

For any particular field, the effect increases until the critical temperature is reached. For temperatures above the

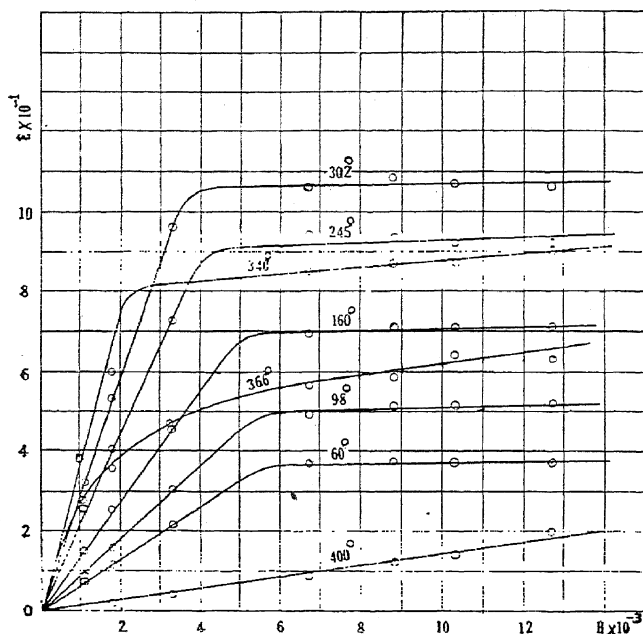


Fig. 109.

critical, 400° C., for example, the Nernst effect in nickel is proportional to the field.

In Fig. 110, we see that the Nernst and Hall effects in *nickel* pass through similar variations with temperature. After the critical temperature has been passed, the two effects fall abruptly, and then go on decreasing as the temperature rises.

The origins for the two curves have been shifted to avoid serious overlapping.

The Nernst effect in *cobalt*, between  $56^{\circ}$  and  $550^{\circ}$  C., was found by Smith to be proportional to the magnetic field up to 12,000 gauss. Since the critical temperature of *cobalt* is about  $1100^{\circ}$  C., the variation of the effect up to and beyond that temperature remains to be determined.

In general, for the temperatures used, the Nernst effect in *cobalt* increases with temperature, and this increase in the Nernst effect is similar to that of the Hall effect as seen from the table below.

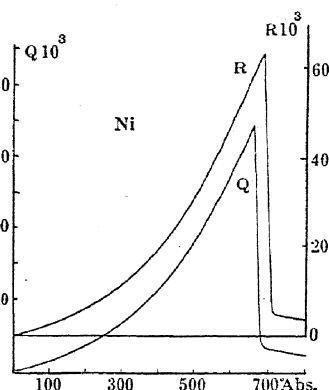


Fig. 110.

Variation of Nernst and Hall Effects with Temperature

Temperature	Nickel		Cobalt	
	$Q$	$R$	$Q$	$R$
300° Abs.	$+5.25 \times 10^{-8}$	$-11.2 \times 10^{-8}$	$+1.80 \times 10^{-8}$	$+5.88 \times 10^{-8}$
350°	7.25	14.8	2.15	7.60
400°	9.75	18.8	2.70	10.0
450°	13.0	24.0	3.30	12.6
500°	16.7	30.2	4.00	16.0
550°	21.3	37.6	4.85	20.0
600°	27.0	46.8	5.70	24.6
650°	34.0	59.0	6.75	29.6
700°	2.0	3.0	7.80	35.0
750°			8.90	40.8
800°			10.00	46.8

*Silicon.* The Nernst effect in *silicon* was observed by Buckley<sup>(96)</sup>, 1914, to decrease with temperature between  $0^{\circ}$  and  $100^{\circ}$  C., in fields up to 20,000 gauss. For all fields the effect was found to be proportional to the field strength. For one specimen,  $Q = -0.020$  at  $50^{\circ}$  C., and  $Q = -0.016$  at  $80^{\circ}$  C.

*Tellurium.* According to Wold <sup>(604)</sup>, 1916, the Nernst effect in *tellurium* increased rapidly to a maximum, at about 100° C., then decreased, and seemed to vanish as the melting

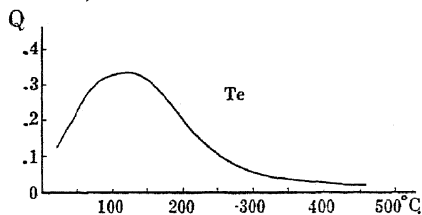


Fig. 111.

point was approached, Fig. 111. Up to a field of about 12,000 gauss, the Nernst effect in *tellurium* was found to be independent of the field strength.

#### 4. Nernst Effect in Alloys

The Nernst effect in *bismuth-tin alloys* was determined in 1887 by Ettingshausen and Nernst <sup>(173)</sup>. The results are given in the table below. The electromotive force  $E$  is in absolute units.

*Nernst Effect in Bismuth-Tin Alloys*

Plate	Bismuth	Tin	$H$	$E$
Bismuth ..	100 %	0 %	2810	— 5070
$L_I$ .....	99.05	0.95	2820	— 2970
$L_{II}$ .....	98.54	1.46	2800	— 2310
$L_{III}$ .....	93.86	6.14	2830	— 324
$L_{IV}$ .....	86.9	13.1	2800	— 128

The Nernst effect in the above alloys falls off rapidly as the tin content is increased.

The Nernst effect in *bismuth-lead alloys* was determined by Van Aubel <sup>(27)</sup>, 1903. The results are given in the table below.

*Nernst Effect in Bismuth-Lead Alloys*

Bismuth	Lead	Galvanometer Deflection
100 %	0 %	4.27
99.43	0.57	9.26
96.44	3.56	7.94

Here we have further evidence of the profound influence of the presence of small amounts of foreign matter, or impurity, upon the Nernst effect in metals.

The Nernst effect in the *Heusler alloy* and its constituents was determined by Zahn and Schmidt <sup>(616)</sup>, 1907. The alloy consisted of about 25 % Mn, 15 % Al, and 60 % Cu. The results are given below for 55° C.

*Nernst Effect in Heusler Alloy*

Metal	$Q$	Metal	$Q$
Heusler Alloy	— 0.000500	Aluminium . .	+ 0.000020
Manganese . .	— 0.000015	Copper . . . .	+ 0.000090

The effect in the alloy is larger than in the component metals. The temperature coefficient of the effect is —0.006. The effect decreased with increase in field strength.

The Nernst effect in *Bi-Sb*, *Sb-Cd*, and *Sb-Zn* alloys was determined by Smith <sup>(487)</sup>, 1911. The effect in the *Bi-Sb* alloys fell off rapidly on the addition of antimony, and soon approached the value of the effect in pure antimony.

In the *Sb-Cd* and *Sb-Zn* alloys the effect fell off gradually at first on the addition respectively of cadmium and zinc; the effect then gradually increased, and reached a maximum singular point when the metals were combined in the ratio of their atomic weights. The effect then decreased and approached the value of the second metal content in its pure state.

The Nernst effect in the magnetic alloys, *monel* and *nichrome*, and in *silicon steel* was investigated by Smith <sup>(490)</sup>, 1913. In monel and nichrome the Nernst electromotive force increased rapidly at first, then more slowly, and finally seemed to approach a limiting value with further increase of the field. Between the temperatures 56° and 167° C., the effect in monel was found to decrease greatly with rise in temperature. In the case of nichrome the effect increased slightly from 95°

to  $205^{\circ}\text{C.}$ , and decreased as the temperature rose to  $308^{\circ}$ . The addition of  $0.25\%$  of silicon to steel reduced the Nernst effect to about one half, while  $3.8\%$  silicon reversed the sign of the effect and increased its magnitude by about  $3\frac{1}{2}$  times.

The following tables contain the Nernst effects determined by Smith <sup>(496)</sup>, 1921, in the magnetic alloys of *nickel-copper* and *iron-nickel*.

*Nernst Effect in Nickel-Copper Alloys*

Ni %	Cu %	$Q$	Ni %	Cu %	$Q$
100	0	+ 0.00348	75	25	+ 0.00350
95	5	+ 0.0118	70	30	+ 0.0174
92.5	7.5	+ 0.0147	65	35	+ 0.00097
90	10	+ 0.0207	50	50	+ 0.00017
85	15	+ 0.0256	—	—	—

*Nernst-Effect in Iron-Nickel Alloys*

Fe %	Ni %	$Q$	Fe %	Ni %	$Q$
100	0	- 0.000975	91.83	8.17	—
98.93	1.07	- 0.000421	89.80	10.20	+ 0.00380
98.07	1.93	- 0.000306	86.89	13.11	+ 0.00495
92.95	7.05	+ 0.001903	44.0	56.0	+ 0.000133

The Nernst effect in the *nickel-copper* alloys reaches a maximum, when the composition points to a compound  $\text{CuNi}_3$ . In the *iron-nickel* alloys, the effect decreases on the addition of nickel, and changes sign when the nickel content is about  $2\%$ .

### 5. Theories of the Nernst Effect

Ettingshausen and Nernst <sup>(172)</sup>, 1886, seemed to think the phenomenon, now known as the Nernst effect, was intimately connected with the molecular structure of the metal.

Nernst <sup>(187)</sup>, 1887, gave the following equations for the effect:

$$u = -k \frac{dp}{dx} + kHQ \frac{dt}{dy},$$

$$v = -k \frac{dp}{dy} - kHQ \frac{dt}{dx}$$

in which  $u$  and  $v$  are the current components along the axis  $x$  and  $y$  respectively,  $t$  the temperature at a point  $(x, y)$ ,  $p$  the potential difference,  $k$  the *electrical* conductivity,  $H$  the magnetic field strength, and  $Q$  the Nernst coefficient. These equations are similar to those developed by Lorentz, p. 76, for the Hall effect.

Riecke <sup>(438)</sup>, 1898, assumed that a heat current in a plate carried with it the two kinds of electric particles, and that the magnetic field acted upon those particles and set up a transverse difference of potential, the coefficient of which is

$$Q = \frac{w_p w_n (v_p + v_n)}{v_p w_n + v_n w_p},$$

in which the letters have the same significance as in the Hall coefficient, p. 79. In the above expression  $Q$  depends upon both the vibratory and the migratory velocities of the two kinds of ions.

The expression deduced by Drude <sup>(149)</sup>, 1900, for the Nernst coefficient is

$$Q = - \frac{p e v_1 v_2 \left( N_1 \frac{dN_1}{dt} + N_2 \frac{dN_2}{dt} \right)}{v_1 N_1 + v_2 N_2},$$

in which the letters have the same meaning as in the Hall coefficient, p. 80. Here  $Q$  is dependent on the velocity of the ions and upon the number of ions per unit volume, which latter is a function of the temperature.

In 1906 Riecke <sup>(440)</sup> developed another expression for the Nernst coefficient, viz:

$$Q = - \frac{1}{16} \frac{v_1 l_1 + v_2 l_2}{T},$$

in which  $v_1$  and  $v_2$  are the velocities, and  $l_1$  and  $l_2$  the mean free paths respectively of the positive and negative ions.

Starting with the supposition of only one kind of free electron, Livens <sup>(329)</sup>, 1915, deduced the following equation for the Nernst electromotive force set up by a magnetic field:

$$E_z = \left( \frac{2}{S} - \frac{1}{2} \right) \frac{\Gamma \left( \frac{3}{2} + \frac{4}{S} \right)}{\Gamma \left( 2 + \frac{2}{S} \right)} \cdot \frac{R l_m q^{\frac{1}{2} - \frac{2}{S}}}{m c} \cdot H \frac{d\Theta}{dy},$$

in which  $S$  is the constant connected with the force exerted by the molecules on the electrons,  $R$  is the usual absolute constant in the equation

$$q = \frac{m}{e R \Theta}.$$

The above equation for  $E$  changes sign according as  $S$  is  $<$  or  $> 4$ . Hence the sign of the Nernst effect may depend on the magnitude of the force acting between the molecules and the electrons.

#### 6. Relation between the Nernst and Righi-Leduc Effects

The magnetic field not only produces a difference of electric potential at the edges of a plate carrying a heat current, but also sets up a difference of temperature at the edges. The latter effect is known as the Righi-Leduc effect, p. 233. If the wires leading from the edges are not of the same material and property as the plate, a thermo-electric E. M. F. will be superposed on the Nernst difference of potential. The Nernst E. M. F.,  $E_Q$ , is found by correcting the observed E. M. F.,  $E_{obs.}$ , as follows:

$$E_Q = E_{obs.} \pm \Delta T \cdot \Theta,$$

where  $\Delta T$  is the difference of temperature set up at the edges of the plate, and  $\Theta$  is the thermo-electric height of the lead wires against the metal of the plate.

Since the true Nernst E. M. F. is given by the equation,

$$E_Q = Q b H \frac{dt}{dl}, \text{ we have } Q b H \frac{dt}{dl} = E_{obs.} \pm \Delta T \cdot \Theta, \text{ or}$$

$$Q = E_{obs.} \div b H \frac{dt}{dl} \pm \Delta T \cdot \Theta \div b H \frac{dt}{dl}.$$

But the Righi-Leduc effect, p. 235, is

$$S = \Delta T \div b H \frac{dt}{dl}.$$

Hence 
$$Q = E_{obs.} \div bH \frac{dt}{dl} \pm S \cdot \Theta.$$

Many of the earlier determinations of the Nernst effect were probably not corrected for the Righi-Leduc difference of temperature.

Zahn <sup>(609)</sup>, 1904, derived the following expression for the ratio of  $Q$  to  $S$ :

$$\frac{Q}{S} = -0.5 \text{ p.c.} \times 10^{-2} \frac{N_2 + N_1}{N_2 - N_1},$$

in which  $p = 1.656 \times 10^{-4}$ ,  $c = 3 \times 10^{10}$ , and  $N_1$  and  $N_2$  the number respectively of negative and positive ions per c.c.

If the above values are substituted and  $N_2$  is considered negligible compared with  $N_1$ , then  $\frac{Q}{S} = 2.5 \times 10^4$ .

The ratios in the following table are due to Zahn (l.c.).

*Ratio of Nernst and Righi-Leduc Effects*

Metal	$\frac{Q}{S}$	Metal	$\frac{Q}{S}$
Bismuth . . . .	$8.7 \times 10^4$	Iron . . . . .	$0.28 \times 10^4$
Antimony . . .	0.88 „	Cobalt . . . . .	1.4 „
Nickel . . . . .	0.65 „		

Roughly speaking, these values are of the same order of magnitude as the theoretical value of the ratio,  $2.5 \times 10^4$ .

Heurlinger <sup>(259)</sup>, 1915, developed the following relation between the adiabatic,  $Q_a$ , and the isothermal,  $Q_i$ , Nernst effects, and the Righi-Leduc effect:

$$Q_a - Q_i = E_i H^2 S,$$

in which  $E_i$  is the coefficient of the thermomagnetic longitudinal potential difference.

## 7. Relation between the Nernst and Hall Effects

Barlow <sup>(44)</sup>, 1903, in his compensation method, sent through *antimony*, *bismuth*, and *zinc* plates both heat and electric currents of such magnitude and in such direction, that the Nernst and Hall E. M. F.'s were equal and of opposite sign.



In this case  $E_{total} = E_R \pm E_Q = 0$ , or

$$E_{total} = \frac{RHI}{d} \pm QbH \frac{dt}{dl} = 0.$$

Hence 
$$\frac{Q}{R} = \pm \frac{I}{bd} \div \frac{dt}{dl}.$$

Since the current density,  $i = \frac{I}{bd}$ ,

$$\frac{Q}{R} = \pm i \div \frac{dt}{dl}.$$

The table below gives the values of  $\frac{Q}{R}$  as determined by Barlow.

Metal	$H$	$\frac{dt}{dl}$	$i = \frac{I}{bd}$	Temperature	$\frac{Q}{R}$
Antimony	9130	10° C.	0.82	44° C.	+0.0817
Bismuth .	9030	12.3°	0.99	50°	-0.081
„	9030	18.5°	1.57	42°	-0.085

The following relation between  $Q$  and  $R$  is Zahn's <sup>(613)</sup> modification of Gans' <sup>(208)</sup> expression:

$$Q = \frac{1}{3} \sigma_0 R_0 \frac{\alpha}{3e} (18 + b - b^2),$$

in which 
$$b = 2T \frac{d \log N}{dT},$$

and  $\sigma_0$  and  $R_0$  are respectively the conductivity and the Hall coefficient in very weak fields, a limitation that applies only to bismuth. If it is admitted that the different signs of  $R$  are only apparent, and that the positive are due to a retrograde inner field, then according to the above equation,  $Q$  and  $R$  will have the same sign in metals in which  $b < 4.77$ , and opposite signs in metals in which  $b > 4.77$ .

Zahn deduced a similar relation between the Nernst effect and the isothermal Hall effect:

$$Q = \frac{\alpha}{24e} \cdot R_i \sigma (15 + 7b).$$

Since  $e$  is negative, the two effects must have opposite signs, unless  $b$  changes sign.

## 8. Relation between the Nernst, Hall, and Thomson Effects

Moreau <sup>(370-2)</sup>, 1900, pointed out the fact that there should exist a simple relation between the Nernst, Hall and Thomson effects, and the specific resistance of a metal.

The Nernst effect equation is

$$Q = \frac{E}{b} \div \frac{dt}{dl} H \dots \dots \dots (1)$$

The static Thomson effect potential gradient is

$$\frac{de}{dl} = \sigma \frac{dt}{dl} \dots \dots \dots (2)$$

where  $\sigma$  is the Thomson effect coefficient.

If equation (1) is divided by (2) we have

$$Q = \left( \frac{E}{b} \div \frac{de}{dl} \right) \frac{\sigma}{H} \dots \dots \dots (3)$$

The usual Hall effect equation may be written

$$R = \frac{E}{b} \div \frac{I}{bd} \cdot H.$$

If  $e$  is the difference of potential at the *ends* of the plate, then the potential gradient is  $\frac{de}{dl}$ , and if  $\varrho$  is the specific resistance of the metal,  $I = \frac{de}{dl} \times \frac{bd}{\varrho}$ .

The Hall equation becomes then,

$$R = \left( \frac{E}{b} \div \frac{de}{dl} \right) \frac{\varrho}{H} \dots \dots \dots (4)$$

If the supposition is made that the equipotential lines of the primary electric current of the Hall effect are rotated by the magnetic field to the same extent as the static thermo-electric equipotential lines of the heat current of the Nernst effect, then the expression in the parenthesis in (3) is equal to the similar expression in (4), and if (3) is divided by (4), we have

$$Q = \frac{\sigma R}{\varrho},$$

which is known as the Moreau equation.

The following table gives the values of  $R$  and  $Q$  as determined by Nernst <sup>(387)</sup>, and the values of  $Q$  computed from

Moreau's formula. The signs of the observed values of  $Q$  are according to the later convention, p. 214.

Metal	$R_{20}$	$\sigma_{20}$	$\varrho_{20}$	$Q_{obs.}$	$Q_{calc.}$
Bismuth .	- 10.1	- 1875	126582	+ 0.196	+ 0.149
Antimony	+ 0.192	+ 2074	44228	+ 0.0094	+ 0.0090
Nickel . .	- 0.024	- 1482	13802	+ 0.0073	+ 0.0026
Cobalt . .	+ 0.0046	- 4073	10675	+ 0.00154	- 0.00175
Iron . . .	+ 0.0113	- 1412	10153	- 0.00156	- 0.00156
Steel . . .	+ 0.0175	- 952	26600	- 0.00060	- 0.00062
Copper . .	- 0.00052	+ 275	1686	- 0.000073	- 0.000084
Zinc . . .	- 0.00041	+ 697	6211	- 0.000054	- 0.000046

The above agreement between the observed and calculated values of  $Q$  is very striking, and seems to point to some fundamental inter-relation between the quantities  $R$ ,  $\sigma$ ,  $\varrho$ , and  $Q$ . Moreau's observed value of  $Q$  for cobalt was -0.00146.

Hall and Campbell<sup>(247)</sup>, 1911, investigated the relation between the above four effects in soft *iron*. The results are given in the table below.

Temperature	$\frac{R}{\varrho}$	$\frac{Q}{\sigma}$
20° C.	+ 686 $\times 10^{-9}$	+ 958 $\times 10^{-9}$
40°	763	934
60°	828	908
80°	884	883
100°	932	858

Here the Moreau formula holds well for 80° C. Lack of better agreement may be due to the fact that all four of the effects were not determined in the same piece of metal.

Voigt<sup>(560)</sup>, 1899, from pure thermodynamic considerations deduced the following relation between the Nernst and Hall effects:

$$Q = -\frac{R}{\varrho} \Phi,$$

in which  $\Phi$  is the thermoelectric power of the metal, against lead probably, although Voigt does not state this fact.

The following table, given by Moreau, shows that the

$Q$ 's of Voigt's formula do not agree, either in magnitude or sign, with the observed values, as well as the Moreau  $Q$ 's do.

Metal	$-\frac{R}{\rho} \Phi$	$\sigma \frac{R}{\rho}$	$Q_{obs.}$
Bismuth . . . .	+ 0.337 or + 0.246	+ 0.149	+ 0.196
Antimony . . .	+ 0.0097	+ 0.0090	+ 0.0094
Nickel . . . . .	+ 0.0039	+ 0.0026	+ 0.0073
Cobalt . . . . .	- 0.00094	- 0.00175	+ 0.00154
Iron . . . . .	+ 0.0018	- 0.00156	- 0.00156
Steel . . . . .	+ 0.00114	- 0.00062	- 0.00060
Copper . . . . .	- 0.000047	- 0.000084	- 0.000073
Zinc . . . . .	- 0.000018	- 0.000046	- 0.000054

From *a priori* considerations, we should expect the Moreau relation to hold better than that of Voigt, since the Hall, Nernst, and Thomson effects are fundamental properties of metals, while the thermoelectric power of a metal is a relative property only.

### 9. Corbino-Nernst Effect

Corbino <sup>(126)</sup>, 1911, observed that a metal disc, heated at the centre, tended to rotate in a magnetic field. This tendency seems to be due to a Nernst difference of potential set up in the plate, that causes an electric current to flow round in the plate.

Corbino observed this effect in *bismuth*, *antimony* and *tellurium*.

Senepa <sup>(478)</sup>, 1912, developed the following expression for the force  $M$ , tending to rotate a metal disc heated at its centre or circumference, and placed in a transverse magnetic field:

$M = m (T_o - T_r) H^2$ , in which  $(T_o - T_r)$  is the difference in temperature between the centre and circumference,  $H$  the magnetic field strength, and

$$m = \frac{d \cdot k \cdot U \cdot S}{4 \log \frac{r_o}{r}},$$

where  $d$  is the thickness of the plate,  $S$  the area of the surface of the disc,  $r$  the radius of the inner circle at the constant lower temperature,  $r_0$  the radius of the outer circle at the higher temperature,  $k$  the thermal conductivity, and  $U$  a constant characteristic of the metal.

His experiments with *antimony* and *bismuth* in fields from 1500 to 2000 gauss supported the theory. Calling  $\bar{U}$  for bismuth 1, he found the constant for antimony to be 0.054. The ratio of the Nernst effects in bismuth and antimony as determined by Nernst is about 0.05.

For the metal disc Tieri <sup>(538)</sup>, 1915, substituted a hollow cylinder. The cylinder was heated by focussing the image of an electric arc lamp on the surface of the cylinder. Cylinders of *bismuth*, *copper*, *silver* and *graphite* were employed.

The rotation effect in *bismuth* in the magnetic field was large, and about 939 times that in *graphite*. No appreciable effect was discovered in *copper* or *silver*.

#### 10. Nernst Effects

The signs of the Nernst effects in these tables are given in accordance with the convention as stated on p. 214, and not that of Nernst, p. 213.

Substance	Temperature	$H$	$Q$	Observer
Aluminium . . .	—	—	+ 0.000042	Unwin
	Room temp.	Ind. H	— 0.000020	Zahn and Schmidt
Antimony . . .	20° C.	"	+ 0.00887	Nernst
	18°	"	+ 0.0176	Zahn
	44°	9130	+ 0.0147	Barlow
Bismuth . . . .	—	—	+ 0.132	Nernst
	42°	10000	+ 0.184	Barlow
" I. . . .	18°	2660	+ 0.125	Zahn
" " . . .	"	3930	+ 0.178	"
" " . . .	"	6290	+ 0.234	"
" " . . .	"	8300	+ 0.261	"
" " . . .	"	10570	+ 0.275	"
	27.5°	3025	+ 0.0533	Lloyd
	"	5100	+ 0.0888	"

Substance	Temperature	$H$	$Q$	Observer
Bismuth . . . .	27.5°	6100	+ 0.112	Lloyd
"	"	6750	+ 0.121	"
"	"	8200	+ 0.133	"
"	"	8925	+ 0.136	"
"	"	9925	+ 0.136	"
"	4°	1900	+ 1.09	"
"	"	3650	+ 0.764	"
"	"	5625	+ 0.648	"
"	"	6950	+ 0.642	"
"	"	8900	+ 0.613	"
"	"	10425	+ 0.552	"
"	- 65° to - 70°	6500	+ 0.517	"
Brass . . . . .	—	—	Negative	Zahn
Cadmium . . . .	—	—	- 0.00012	Unwin
Carbon . . . . .	—	—	(+ 0.0001)	Nernst
"	—	—	Not detected	Zahn
Cerium . . . . .	60°	17000 to 18000	+ 0.00010	Smith, A. W.
Cobalt . . . . .	Room temp.	—	+ 0.00224	Nernst
"	"	—	+ 0.00154	Ettingshausen and Nernst
"	18°	6290	+ 0.0020	Zahn
"	"	10570	+ 0.0018	"
"	60°	17000 to 18000	+ 0.00200	Smith, A. W.
"	—	—	- 0.00146	Moreau
"	—	—	+ 0.00078	Unwin
Constanstan . .	—	—	Not detected	Zahn
Copper . . . . .	Room temp.	—	- 0.000090	Nernst
"	"	—	- 0.000073	Ettingshausen and Nernst
"	—	—	- 0.00019	Unwin
"	18° C.	—	- 0.00027	Zahn
German Silver .	"	—	Not detected	"
Heusler Alloy .	55°	3400	+ 0.00057	Zahn and Schmidt
"	"	5100	+ 0.00048	"
Indium . . . . .	60° C.	Ind. H	+ 0.000032	Smith, A. W.
Iridium I. . . .	—	—	- 0.000005	Zahn
" II. . . . .	—	—	> - 0.000002	"
Iron . . . . .	—	—	- 0.00156	Nernst
"	48°	10800	- 0.00087	Campbell
"	55°	6290	- 0.00105	Zahn
"	31°	5550	- 0.000860	Halland Campbell

Substance	Temperature	$H$	$Q$	Observer
Iron. . . . .	60°	5500	- 0'000980	Hall and Campb
	60°	17000 to 18000	- 0'000810	Smith, A. W.
	—	—	- 0'000095	Unwin
	—	—	- 0'000646	Moreau
Lead . . . . .	—	—	- 0'000005 (?)	Nernst
Lithium . . . .	60°	Ind. H	+ 0'000160	Smith, A. W.
Manganese . . .	—	—	+ 0'000015	Zahn and Schmid
Molybdenum . .	60°	Ind. H	- 0'000172	Smith, A. W.
Monel . . . . .	56°	1500	+ 0'323	"
Nichrome . . .	43°	2500	+ 0'0708	"
Nickel . . . . .	56°	—	+ 0'00861	Nernst
	—	—	+ 0'0073	Ettingshausen and Nernst
Nickel I. . . . .	55°	10620	+ 0'0013	Zahn
" II. . . . .	"	6290	+ 0'00355	"
	27°	—	+ 0'00525	Smith, A. W.
	37°	1700	+ 0'0038	Moreau
	"	13800	+ 0'0015	"
	—	—	+ 0'0010	Unwin
Palladium I. . .	—	—	+ 0'000127	Zahn
" II. . . . .	—	—	+ 0'000051	"
Platinum . . . .	—	—	Not detected	"
Silicon . . . . .	47'4° C.	5800	+ 0'0333	Gottstein
	47'8°	5000	+ 0'029	"
" I. . . . .	50°	—	+ 0'020	Buckley
" . . . . .	80°	—	+ 0'016	"
" II. . . . .	50°	—	+ 0'0113	"
Silver . . . . .	—	—	- 0'000046	Nernst
	—	—	- 0'00043	Zahn
	—	—	- 0'00018	Unwin
Steel . . . . .	—	—	- 0'000706	Nernst
	55° C.	—	- 0'00166	Zahn
	—	—	- 0'000596	Moreau
Tantalum . . . .	60°	Ind. H	+ 0'000098	Smith, A. W.
Tellurium . . .	33°	3000	+ 0'36	Lloyd
	45'8°	4340	+ 0'287	Wold
Thallium . . . .	60°	Ind. H	- 0'000037	Smith, A. W.
Tin . . . . .	—	—	- 0'000004 (?)	Nernst
Tungsten . . . .	60°	Ind. H	- 0'00100	Smith, A. W.
Zinc . . . . .	—	—	- 0'000054	Nernst
	—	—	- 0'000240	Zahn
	—	—	- 0'000073	Unwin

## CHAPTER XIII

THE RIGHI-LEDUC EFFECT: TRANSVERSE  
THERMOMAGNETIC TEMPERATURE DIFFERENCE

## 1. Historical and General

In their thermomagnetic investigations, Ettingshausen and Nernst <sup>(172)</sup>, 1886, endeavored to find out whether the electromotive force, set up by the magnetic field at the edges of a plate carrying a heat current, was a thermoelectric one. They replaced the copper lead wires at the edges of the plate with copper-German silver thermocouples, but failed to detect any change in temperature.

In 1887, Righi <sup>(447)</sup>, and Leduc <sup>(318)</sup>, independently and practically simultaneously, discovered that the magnetic field did produce a change in temperature at the edges of a plate through which a heat current was flowing.

Righi <sup>(447)</sup>, 1887, measured the change in temperature produced in a plate of *bismuth* by the magnetic field at three points, *a*, *b*, *c*, in the plate *P*, Fig. 112.

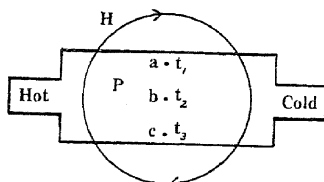


Fig. 112.

The following table gives the results of his measurements.

No Magnetic Field			Field as in Fig. 112			Field Reversed		
$t_1$	$t_2$	$t_3$	$t_1$	$t_2$	$t_3$	$t_1$	$t_2$	$t_3$
9.09° C.	9.617°	9.067°	8.97°	9.575°	9.063°	8.985°	9.54°	8.90°

When the magnetic field was on, as in Fig. 112, the point *a* was cooled, and when the field was reversed the point *c* was cooled. That is, the isothermal lines were rotated.

Leduc <sup>(318)</sup>, 1887, while studying the effect of the magnetic field on the thermal conductivity of *bismuth*, also found that the isothermals of the plate were rotated.



On the other hand, the effect is said to be *negative*, as in *bismuth*, when the passage from the entrance *F*, Fig. 116, to the warmer edge of the plate, *A*, is opposed to the direction of the magnetizing current, *H*.

The change in temperature at the edges of the plate is usually determined by means of thermo-couples, *t*, *t*, as indicated in Figs. 115 and 116.

### 3. Angle of Rotation of the Isothermal Line

If the distance *AC*, through which one end of the isothermal line *AB*, Fig. 115, is moved by the magnetic field, is called *x*, then the tangent of the angle of rotation *d* is given by the relation:

$$\tan d = \frac{2x}{b},$$

in which *b* is the width of the plate.

$$\text{But} \quad \frac{\Delta T}{2x} = \frac{dt}{dl}.$$

$$\text{Hence} \quad \tan d = \Delta T \div b \frac{dt}{dl} = S \cdot H.$$

The tangent of the angle of rotation then depends directly on the difference of temperature at the edges, and indirectly on the width and temperature gradient of the plate.

Under the Hall effect, p. 12, we saw that the angle of rotation of the equipotential line is given by the equation,

$$\tan \Phi = R \sigma H.$$

The following table according to Zahn <sup>(611)</sup>, 1908, gives the values of *d* and  $\Phi$  for several metals, when *H* = 10,000 gauss.

*Rotation of Equipotentials and Isothermals*

Metal	$\Phi$	<i>d</i>	Metal	$\Phi$	<i>d</i>
Te . . .	+ 3° 8'	+ 2° 16'	Pt . . .	— 0° 6'	— 0° 7'
Sb . . .	+ 2° 59'	+ 1° 9'	Pd . . .	— 1° 0'	— 0° 9'
Steel . .	+ 20' to 6'	+ 24'	Cu . . .	— 8° 9'	— 7° 9'
Fe . . .	+ 22'	+ 13'	Ag . . .	— 18'	— 14'
Co . . .	+ 47'	+ 3° 8'	Ni . . .	— 20'	— 11'
Zn . . .	+ 6° 2'	+ 4° 4'	Bi . . .	— 24° 30'	— 1° 10'
Ir . . .	+ 1° 4'	+ 1° 7'			

The above values for the ferromagnetic metals are those for weak fields.

The angles  $\Phi$  and  $d$  agree fairly well in magnitude for all the above metals, except for *bismuth*, which exhibits many abnormalities in all the effects herein treated. In all cases the corresponding angles have the same sign, and the Hall and Righi-Leduc effects have the same sign, except in some specimens of bismuth, in which the effect changes sign with field and temperature.

#### 4. Correction of the Righi-Leduc Effect for Thermal Conductivity

The Righi-Leduc equation,

$$S = \Delta T \div Hb \frac{dt}{dl},$$

may be written 
$$S = \frac{\Delta T \cdot k \cdot d}{A \cdot H},$$

where  $k$  is the thermal conductivity of the metal,  $d$  the thickness of the plate, and  $A$  is equal to  $kbd \frac{dt}{dl}$ , or the quantity of heat flowing through the plate.

Evidently some of the heat will be lost at the edges of the plate to the surrounding material, and, as Zahn<sup>(609)</sup> has pointed out, a correction must be made for this loss. The correction factor is similar to that used for the correction of the Ettingshausen effect, p. 149:

$$S_{corr.} = \left\{ \frac{1}{2} \beta \frac{(u-v)}{e^{b\beta} - e^{-b\beta}} \right\} \Delta T \div Hb \frac{dt}{dl}.$$

Zahn applied this correction in the case of *antimony*, with the following results:

Antimony	$S$ uncorrected	$S$ corrected
Plate I . . . . .	$18.2 \times 10^{-7}$	$20.1 \times 10^{-7}$
" II . . . . .	18.7 "	19.4 "

In the case of the first plate the correction factor is 1.104, while in the second it is 1.039. Hence, unless a cor-

rection is made for heat loss, the value of the Righi-Leduc effect may be from 4% to 10% too small.

### 5. Relation between the Righi-Leduc and Hall Effects

If we suppose that the Righi-Leduc isothermal and the Hall equipotential lines are rotated by the magnetic field through the same angle, we shall have

$$\tan d = S \cdot H = \tan \Phi = R \sigma H.$$

Hence  $S = \sigma R$ , where  $\sigma$  is the electrical conductivity of the metal.

The table below shows that this relation holds only approximately.

Metal	$\sigma \times 10^5$	$R \times 10^4$	$\sigma R \times 10^8$	$S \times 10^8$
Nickel . .	12	-100	-126	-44
Silver . . .	62	-9	-55	-41
Palladium	3.3	-9	-3	-3
Copper . .	58	-4	-23	-18
Platinum .	7	-1.3	-1	-2
Iridium . .	12.5	+3.8	+5	+5
Zinc . . .	17	+10	+17	+13
Cobalt . .	10	+16	+16	+13
Iron . . .	6.7	+110	+74	+39
Antimony	2.3	+2000	+460	+200
Tellurium	0.00078	+6000000	+468	+400

Königsberger and Gottstein<sup>(297)</sup>, 1915, have shown that according to the theory of Gans<sup>(268)</sup>, the following relation should hold:

$$S = \frac{7}{8} \frac{R_i}{r},$$

where  $R_i$  is the isothermal Hall effect, and  $r$  is the specific resistance of the metal.

### 6. Variation of the Righi-Leduc Effect with Magnetic Field and Temperature

Senepa<sup>(479)</sup>, 1913, measured the Righi-Leduc effect in *bismuth* in fields from 600 to 1200 gauss. The mean temperature was 43.3° C. The results are plotted in

Fig. 117. The effect is independent of the field up to about 1000 gauss.

The curves in Fig. 118 give the variation of the Righi-Leduc effect in *iron*, *nickel* and *cobalt*, according to the measurements of Smith and Smith<sup>(497)</sup>, 1915.

The effect in each metal was found proportional to the magnetization, and approached a saturation value.

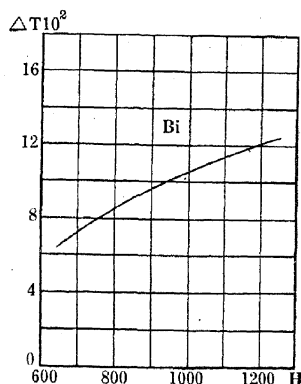


Fig. 117.

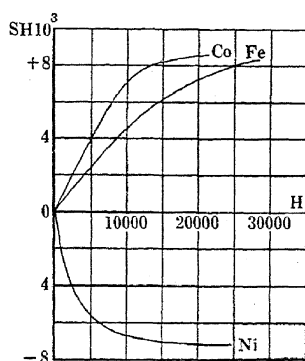


Fig. 118.

Zahn<sup>(609)</sup>, 1904, found that in *bismuth* the Righi-Leduc effect had a small positive temperature coefficient.

Hall and Campbell<sup>(247)</sup>, 1911, determined the Righi-Leduc effect in soft *iron* at two mean temperatures, with the following result:

Righi-Leduc Effect in Iron

Metal	$H$	Temperature	$S$
Iron. . . . .	5600	30.6° C.	$+5.68 \times 10^{-7}$
" . . . . .	5500	60.2	$+6.32$ "

In this case the effect increases with temperature, and the temperature coefficient is  $+0.0038$ .

The following table gives the Righi-Leduc effect in *tellurium*, as determined by Wold<sup>(604)</sup>, 1916, at 114° C.

*Righi-Leduc Effect in Tellurium*

<i>H</i>	<i>S</i>	<i>H</i>	<i>S</i>
3,120	$+6.5 \times 10^{-8}$	8,920	$+6.5 \times 10^{-8}$
6,390	5.7 "	10,700	5.8 "
7,750	6.2 "		

The effect is practically independent of the field.

## 7. The Righi-Leduc Effect in Alloys

Smith and Smith <sup>(497)</sup>, 1915, determined the Righi-Leduc effect in *bismuth-antimony*, and in *antimony-zinc alloys*, in magnetic fields of about 11,000 gauss, and at temperatures at the middle of the plates between 50° and 60° C.

The results are given in the tables below. It will be seen that the addition of *antimony to bismuth* decreased the effect, but not so rapidly as called for by the additive law. According to this law, since the effect is negative in bismuth and positive in antimony, the effect in the alloy should be zero when the antimony content is 61%. The effect, however,

*Righi-Leduc Effect in Bismuth-Antimony Alloys*

Bi %	Sb %	<i>H</i>	<i>S</i> × 10 <sup>8</sup>	Bi %	Sb %	<i>H</i>	<i>S</i> × 10 <sup>8</sup>
100	0	10,900	-405	39	61	12,000	-60
91	9	10,800	-375	30	70	11,100	+9.5
70	30	11,000	-354	20	80	10,800	+95
60	40	"	-195	10	90	12,060	+132
50	50	"	-154	0	100	11,000	+262

*Righi-Leduc Effect in Antimony-Zinc Alloys.*

Sb %	Zn %	<i>H</i>	<i>S</i> × 10 <sup>8</sup>	Sb %	Zn %	<i>H</i>	<i>S</i> × 10 <sup>8</sup>
100	0	11,000	+262	50	50	10,900	8.50
90	10	10,900	42.7	40	60	"	6.77
84	16	"	32.0	30	70	"	8.40
80	20	"	27.5	16	84	"	6.60
70	30	"	20.9	0	100	"	12.4
60	40	"	9.08				

becomes zero only after the percentage of antimony has reached about 67.

In the *antimony-zinc alloys* the effect falls off very rapidly at first, then slowly falls to a minimum, and is followed by a slight increase.

### 8. Theories of the Righi-Leduc Effect

The expression for the Righi-Leduc effect according to the dualistic theory of Riecke <sup>(438)</sup>, 1898, is:

$$\Delta T = \frac{v_p v_n (w_p - w_n)}{v_p w_n + v_n w_p} \cdot Hb \frac{dt}{dl}.$$

According to this equation the effect depends on the translatory and vibratory velocities of the positive and negative particles.

According to Drude <sup>(149)</sup>, 1900, the effect is given by the equation,

$$\Delta T = \frac{e v_p v_n (N_p - N_n)}{c (v_p N_p + v_n N_n)} \cdot Hb \frac{dt}{dl},$$

in which the effect depends on the translatory velocities of the two kinds of particles, and upon the number of the particles per unit volume.

In 1906 Riecke <sup>(440)</sup> deduced this expression for the Righi-Leduc effect:

$$S = \frac{e}{8 \left( 1 + \frac{4}{3} T \frac{d \log n_1 n_2}{dT} \right)} \cdot \frac{v_1 l_1 - v_2 l_2}{c \alpha T}$$

in which the letters have the same significance as in the Ettingshausen effect, p. 151.

From the expressions deduced by Riecke <sup>(440)</sup> in 1906, the following relation is obtained for the Hall, Ettingshausen, Nernst, and Righi-Leduc effects:

$$\frac{R}{P} \cdot \frac{Q}{S} = - \left( 1 + \frac{4}{3} T \frac{d \log n_1 n_2}{dT} \right)^2 \cdot \frac{e^2 \alpha^2}{4 e^2},$$

in which the factors have their usual significance.

The expression developed for the Righi-Leduc effect by Livens <sup>(329)</sup>, 1915, based on the unitary free electron theory, is as follows:

$$\frac{d\Theta}{dz} = -\frac{(7S^2 + 8S + 16)}{8S(S+1)} \cdot \frac{\Gamma\left(\frac{3}{2} + \frac{4}{S}\right)}{\Gamma\left(2 + \frac{2}{S}\right)} \cdot \frac{el_m q^{\frac{1}{2} - \frac{2}{S}}}{mc} \cdot H \frac{d\Theta}{dy},$$

in which the terms have the same significance as in the expression for the Hall and other effects deduced by Livens.

In the above expression the sign of the effect cannot change with the magnitude of  $S$ . Hence the sign of the Righi-Leduc would seem not to depend on the magnitude of the force acting between the electrons and atoms, since  $S$  is proportional to this force.

With the exception of the erratic metal bismuth, in which the two effects may be either positive or negative, the Righi-Leduc and the Hall effects have the same sign in all metals thus far examined. This implies that the isothermal and equipotential lines are rotated in the same direction by the magnetic field, and points to the same fundamental influence exerted by a magnetic field on the movement of electrons and atoms, both when they are carriers of a heat and an electric current.

Unwin<sup>(551b)</sup>, 1920, determined the Hall, Ettingshausen, Nernst, and Righi-Leduc effects in the same specimens of Ag, Al, Cd, Co, Cu, Fe, Ni, and Zn, and found that the ratios  $\frac{R}{S}$ ,  $\frac{Q}{P}$ ,  $\frac{R}{P}$ , and  $\frac{Q}{S}$  were of the same order of magnitude for all of the eight metals.

*Righi-Leduc Effects*

Substance	Temperature	$H$	$S$	Observer
Aluminium . . .	—	—	$-0.62 \times 10^{-7}$	Unwin
Antimony . . .	44° C.	9,130	$+2 \times 10^{-6}$	Barlow
	18°	Ind. H	$+2.01$ "	Zahn
	56.7°	11,000	$+2.62$ "	Smith and Smith
Bismuth I . . .	42°	10,000	$+1.1$ "	Barlow
" II . . .	—	—	Negative	"
" I . . .	18°	3,930	$-2.05 \times 10^{-6}$	Zahn
" " . . .	—	6,290	$-1.12$ "	"
" " . . .	—	8,300	$-0.55$ "	"

Substance	Temperature	$H$	$S$	Observer
Bismuth I . . .	—	10,570	$-0.19 \times 10^{-6}$	Zahn
" II . . .	—	6,290	$+1.74$ "	"
" " . . .	—	8,300	$+1.82$ "	"
" " . . .	—	9,380	$+1.86$ "	"
" " . . .	—	10,570	$+1.92$ "	"
Bismuth . . . .	$34^{\circ}$	6,100	$-1.6$ "	Everdingen
	$43.3^{\circ}$	300 to 1,200	$-2.30$ "	Senepa
	$52^{\circ}$	10,900	$-4.05$ "	Smith and Smith
Cadmium . . . .	$60.5^{\circ}$	12,200	$+1.1 \times 10^{-7}$	"
	—	—	$+0.89$ "	Unwin
Cobalt . . . . .	—	10,570	$+1.3$ "	Zahn
	$53.7^{\circ}$	11,800	$+6.4$ "	Smith and Smith
	—	—	$+1.1$ "	Unwin
Copper I . . . .	—	Ind. H	$-1.4$ "	Zahn
" II . . . . .	—	"	$-2.32$ "	"
	—	—	$-2.1$ "	Unwin
Heusler Alloy .	—	—	Positive	Zahn and Schmidt
Iridium I . . . .	—	—	$+5.5 \times 10^{-8}$	Zahn
" II . . . . .	—	—	$+4.1$ "	"
Iron (soft) . . .	$48^{\circ}$ C.	7,300	$+6.08 \times 10^{-7}$	Campbell
" I . . . . .	$44^{\circ}$	6,500	$+4.29$ "	Hall and Campbell
" II . . . . .	$60.2^{\circ}$	5,500	$+6.32$ "	"
	—	6,290	$+3.9$ "	Zahn
" (electrolytic)	$56.9^{\circ}$	11,900	$+3.92$ "	Smith and Smith
	—	—	$+5.2$ "	Unwin
Steel . . . . .	—	—	$+6.87$ "	Zahn
Molybdenum . .	$57.5^{\circ}$	12,300	$+1.75$ *)	Smith and Smith
Nickel (electrolytic)	—	10,620	$-2.0$ "	Zahn
	—	6,290	$-5.5$ "	"
	$61^{\circ}$	10,880	$-6.15$ "	Smith and Smith
	—	—	$-2.5$ "	Unwin
Nichrome . . .	$59.3^{\circ}$	10,800	$+3.88$ "	Smith and Smith
Palladium I . .	—	—	$-3.3 \times 10^{-8}$	Zahn
" II . . . . .	—	—	$-1.8$ "	"
Platinum . . . .	—	—	$-2.1$ "	"
Silicon . . . . .	$50^{\circ}$	—	$< 5$ "	Buckley
	$47.8^{\circ}$	5,000	$+2.5 \times 10^{-7}$	Gottstein
	$47.4^{\circ}$	5,800	$+2.97$ "	"
	—	—	$+3.2$ "	Königsberger and Gottstein
Silver . . . . .	—	—	$-4.04$ "	Zahn
	—	5,200	$-2.7$ "	Unwin
Tellurium . . .	$30^{\circ}$ to $38^{\circ}$	7,750	$+4 \times 10^{-8}$	Lloyd
	$114^{\circ}$	12,200	$+6.2$ "	Wold
Tungsten . . . .	$58.6^{\circ}$	10,900	$+1.5 \times 10^{-7}$	Smith and Smith
Zinc . . . . .	$58.5^{\circ}$	—	$+1.24$ "	"
	—	—	$+1.29$ "	Zahn
	—	—	$+1.1$ "	Unwin

\*) In a letter to the author, April 25, 1921, the observers state that the sign of  $S$  in molybdenum should be positive, and not negative as printed in the original paper (497).



## CHAPTER XIV

## 1. Thermomagnetic Longitudinal Potential-Difference

Ettingshausen and Nernst <sup>(172)</sup>, 1886, discovered that a transverse magnetic field set up a longitudinal electromotive force in a plate of *bismuth*, one end of which was hot and the other cold. The effect seemed to vary with the square

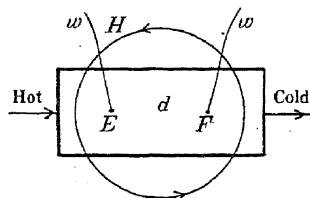


Fig. 119.

of the magnetic field strength and with the temperature gradient, but was independent of the dimensions of the plate. If, in Fig. 119,  $d$  is the distance between the points  $E$  and  $F$ , the difference of whose potential  $\Delta E$  is sought, then the coefficient of the longitudinal potential difference is given by the equation,

$${}_hL_e = \pm \frac{\Delta E}{H^2 \cdot d \cdot \frac{dt}{dl}},$$

where  $\frac{dt}{dl}$  is the temperature gradient.

The above equation may be written

$${}_hL_e = \pm \frac{\Delta E}{H^2 (t_2 - t_1)},$$

where  $t_2$  and  $t_1$  are respectively the temperatures at the points  $E$  and  $F$ .

In the above equation the effect is said to be *positive* if the longitudinal E.M.F. would send an electric current in the same direction as the heat current.

Nernst <sup>(387)</sup>, 1887, detected this longitudinal E. M. F. in several specimens of *bismuth*. In No. I the effect was positive, in No. III negative, while in No. II the effect was negative when  $(t_2 - t_1)$  was  $18.1^\circ$ , and positive when  $(t_2 - t_1)$  was  $86.2^\circ$  C.

The following table gives the values of the effect in plate No. I.

$H$	$\Delta E$	$L$	
2826	600	+ 0.0005147	$(t_2 - t_1) = 1.46^\circ \text{C.}$
6090	1860	+ 0.0003435	Mean temp.
11490	4420	+ 0.0002293	about $10^\circ \text{C.}$
6120	17350	+ 0.0001653	$(t_2 - t_1) = 28^\circ \text{C.}$
11580	50900	+ 0.0001149	Mean temp. about $30^\circ \text{C.}$

From the above results, it is seen that the coefficient of the effect,  $L$ , decreases for higher fields and for higher mean temperature. Nernst sought in vain for the effect in *antimony*, *silver*, *zinc*, and *German silver*. In *alloys of bismuth* containing 1% and 3% of *tin*, the effect was positive, and larger in the specimen that had a content of 1% *tin*.

Ettingshausen<sup>(169)</sup>, 1887, found that the direction of the thermomagnetic longitudinal E. M. F. in very pure *bismuth* depended upon the mean temperature of the portion of the plate examined. In a field of 9500 gauss the longitudinal E. M. F. between two points on the plate at  $99^\circ$  and  $56^\circ \text{C.}$  was 39 microvolts, and negative. Between two points at  $56^\circ$  and  $36^\circ \text{C.}$  the E. M. F. was 40 microvolts, and positive. Between points at  $36^\circ$  and  $24^\circ \text{C.}$ , and between points at  $24^\circ$  and  $20^\circ \text{C.}$ , the effect was positive.

Ettingshausen obtained the same E. M. F. with *copper* as with *German silver electrodes* against the *bismuth* plate. Since the thermoelectric powers of Bi-Cu and Bi-German silver are to each other as 6:5, the magnetic field must exert the same change in the two combinations of metals. Grimaldi<sup>(223)</sup>, 1887, remarked that the thermomagnetic longitudinal potential difference was due to the change in the thermal conductivity and in the thermo-electric power produced by the magnetic field.

Defregger<sup>(141)</sup>, 1897, attached both *copper* and *bismuth leads* to the *bismuth* plate in which he sought to measure the longitudinal E. M. F. In the table below are given the mean galvanometer deflections, which are proportional to the

E. M. F.'s. The effect was found proportional to the first power of the field.

$H$	Copper Leads		Bismuth Leads	
	Deflection	Difference	Deflection	Difference
0	97.2	} 13.4	15.0	} 2.6
+ 11,500	110.6		17.6	
0	98.0	} 19.2	14.8	} 3.2
- 11,500	117.2		18.0	

According to the above, the nature of the leads from the plate exerts a profound influence on the magnitude of the measured effect.

Van Everdingen <sup>(182)</sup>, 1899, determined the longitudinal thermomagnetic E. M. F. in *electrolytic bismuth* in terms of the increase in the thermoelectric power of bismuth against the copper leads in the magnetic field. The results are given below.

$H$	Per cent. Increase in Thermo-E. M. F.	
	Observed	Calculated
2000	1.84	1.81
3000	3.55	3.52
4000	5.52	5.51
5000	7.63	7.67
6000	10.00	9.98

The values in the last column are calculated from an empirical formula derived by Everdingen:

$$E = \frac{C_2 H^2}{1 + C_1 \sqrt{H^2}},$$

in which  $H$  is in kilogauss.

The effect was found to vary with the field strength in the same way that the resistance varied.

Lownds <sup>(343)</sup>, 1901, measured the thermomagnetic longitudinal E. M. F. in a *bismuth wire*, whose ends were successively at temperatures ranging from that of boiling water to that of liquid air. The longitudinal E. M. F. was found by

a null compensation method. First the electromotive force,  $E_1$ , was found when no magnetic field was on, then  $E_2$  when the field was on,  $E_3$  when the field was reversed, and finally  $E_4$ , when the field was taken off. Then the longitudinal E. M. F. set up by the magnetic field is

$$E = \frac{(E_2 + E_3) - (E_1 + E_4)}{2}.$$

The curves in Fig. 120 represent the variation of the

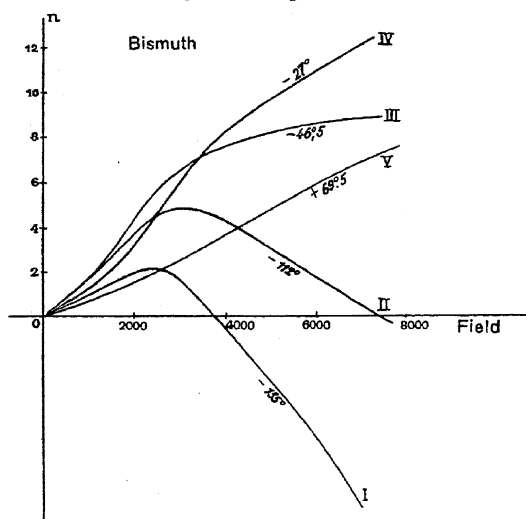


Fig. 120.

effect with field strength and with temperature. The E. M. F. per  $1^\circ$  difference in temperature is given by  $n = \frac{E}{t_2 - t_1}$ , where  $t_2$  and  $t_1$  are the temperatures of the junctions of the leads.

Lownd's convention for the sign of the effect is just opposite to that of Nernst, p. 244.

The curves I, III, V give the change in the effect in a *bismuth plate* at different temperatures, while curves II and IV represent the variation of the effect in *bismuth wire*. In every case the effect falls off in higher fields, and in two cases reverses sign.

Lownds <sup>(344)</sup>, 1901, also measured the longitudinal thermomagnetic E. M. F. in a *crystal of bismuth*, so cut that the principal axis was in the plane of the plate and parallel to its length. The longitudinal effect was measured both when the flow of heat was parallel and when perpendicular to the principal axis. In the neighborhood of  $5^{\circ}$  C. the effect was positive when the heat flow was parallel to the principal axis, and negative when perpendicular to the axis. At about  $-95^{\circ}$  C. the effect for each type of flow of heat was negative.

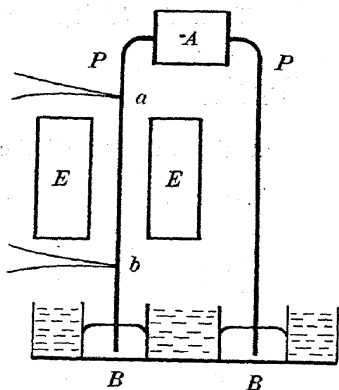


Fig. 121.

Moreau <sup>(374)</sup>, 1901, measured the longitudinal thermomagnetic E. M. F. in *nickel, soft iron, and soft steel*. The pieces of metal were bent into a U-shape, and inverted as indicated in Fig. 121. The ends of the plates, *PP*, dipped

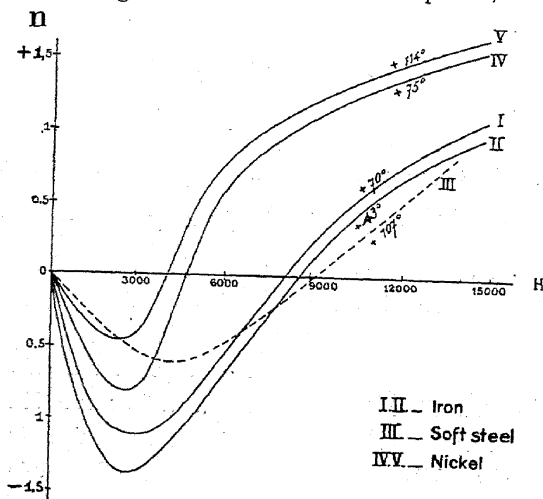


Fig. 122.

into mercury at *BB*. One branch of the plate was placed between the poles *E, E*, of an electromagnet. Thermocouples of copper and platinum were soldered to the plate at *a* and *b*.

The plate was heated through a copper block *A*. The sign and magnitude of the effect depended on the temperature gradient, the mean temperature and the magnetic field, as shown by the curves in Fig. 122. In all cases the effect reaches a maximum, changes sign, and increases in higher fields. The ordinate *n* is the galvanometer deflection divided by the difference in temperature at *a* and *b*, Fig. 121.

The sign for the effect employed by Moreau and Lownds is the opposite of Nernst's convention, p. 244.

Lloyd <sup>(332)</sup>, 1901, observed a longitudinal thermomagnetic E. M. F. in *tellurium*. The warmer end of the plate was found to be at the higher potential.

Barlow <sup>(44)</sup>, 1903, studied the longitudinal thermomagnetic effect in *antimony* and *bismuth*. In antimony no appreciable effect could be detected. In bismuth the effect was observed both in a transverse and in a longitudinal magnetic field.

Curve I, Fig. 123, gives the variation of the effect in a transverse field, while curve II gives the change in a longitudinal field. The latter effect is about one-half the former.

Barlow's convention for sign is opposite that of Nernst.

Zahn <sup>(609)</sup>, 1904, sought for the above effect in a number of metals, but found it only in *bismuth*. In two plates; in fields from 2000 to 16,000 gauss, the E. M. F. was opposed to the direction of the heat current, while in a third plate it was in the direction of the current. The effect was approximately proportional to the first power of the field strength.

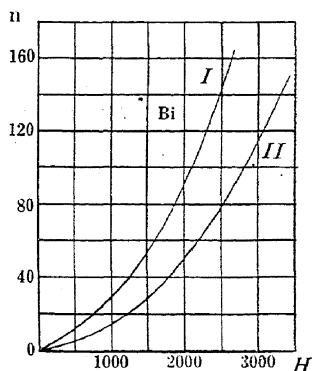


Fig. 123.

Hall and Campbell <sup>(247)</sup>, 1911, measured the effect in soft iron, and obtained these results.

$H$	Mean Temperature	$hL_e$
2000	47° C.	$211 \times 10^{-5}$
8200	"	83 "
12000	"	80 "

The effect was found to be positive, according to Nernst's convention. The effect decreased with rise in field strength, and seemed to approach a saturation value.

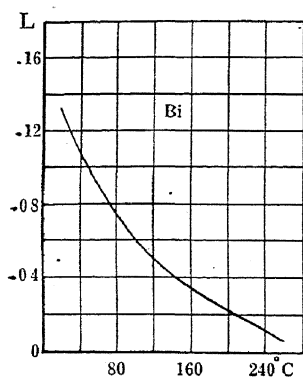


Fig. 124.

Smith <sup>(492)</sup>, 1913, measured the longitudinal thermomagnetic E. M. F. in *bismuth*, through a wide range in temperature. The potential gradient was opposed to the temperature gradient. The curve in Fig. 124 gives the variation of the effect with temperature, in a field of 9600 gauss. The curve seems to indicate that the effect disappears at the melting point of bismuth.

The table below gives the coefficient  $L$  for *bismuth* as determined by several observers.

$H$	Temperature	$L$	Observer
9600	53° C.	— 0'102	Smith, A. W.
11000	"	— 0'100	Barlow
6290	24'1° C.	— 0'111	Zahn
9600	"	— 0'128	Smith, A. W.

The agreement above is unusual, since the effect was measured in different specimens of the metal.

The following table gives the effect for *graphite*, determined by Smith <sup>(492)</sup>.

$H$	$L \times 10^4$	$H$	$L \times 10^4$
3300	+ 65	15600	+ 173
5150	80	18400	193
9300	121	20200	197
12850	151		

This increase in  $L$  with the increase in field strength is analogous to the increase in the resistance of *graphite* in the field, observed by Roberts <sup>(454)</sup>.

Smith also determined  $L$  for the alloy *monel* and for alloys of *bismuth* and *antimony*. In *monel* the effect was negative. The table below gives the values of  $L$  for *monel* (68% Ni, 29.5% Cu, 1.5% Fe, 1% Mn).

$H$	$L \times 10^5$	$H$	$L \times 10^5$
3500	- 484	13000	- 406
5200	498	18500	327
9300	448	20000	306

In the alloys of bismuth and antimony,  $L$ , as well as the thermoelectric height, passed through a maximum when the alloy contained 9% of antimony.

## 2. Theory of the Longitudinal Thermomagnetic Potential-Difference

### *General Considerations*

Grimaldi <sup>(223)</sup>, 1887, attributed the longitudinal thermomagnetic potential difference to the change in thermal conductivity and thermoelectric power of the metal in a magnetic field. A change in thermal conductivity would bring about a change of temperature at the points where the junctions are fastened to the plate, and a difference in thermo-electromotive force would follow.

Since the magnetic field changes the inherent thermoelectric power of the plate or the lead wire, a difference of potential will be set up. There seems to be good reason to believe that both of the above changes are concerned in the production of the longitudinal E. M. F



Defregger <sup>(141)</sup>, 1897, by the use of bismuth leads, seems to have proved that the effect in *bismuth* is not wholly due to changes in thermal conductivity and thermoelectric power. It may be, however, that the magnetic field sets up different changes in the bismuth-bismuth junctions at the two different temperatures at the points of contact.

Lownds <sup>(343)</sup>, 1901, claimed to have demonstrated that the longitudinal E. M. F., which he measured, was not due to an alteration in the thermoelectric power of the metal.

Most investigators have assumed that this effect is due to the change in thermoelectric power in the magnetic field, and have computed their results from the relation

$$n = \Delta \Theta = \Theta_H - \Theta_0,$$

where  $n = \frac{\Delta E}{t_2 - t_1}$ , as before, and  $\Theta_H$  the thermoelectric height in the field  $H$ , and  $\Theta_0$  the thermoelectric height in zero field.

If (as Lownds did) the E. M. F.'s are measured by a null compensation method, any error due to the Peltier effect is eliminated. And if the E. M. F. is measured before, when, and after the field is put on, and there is an algebraic difference between the "ons" and "offs", or  $(E_2 + E_3) - (E_1 + E_4)$ , is not zero, then  $\Delta E$  is not wholly due to a change in thermoelectric power, unless the metal resumes its original properties when the field is removed.

### 3. Thermomagnetic Longitudinal Potential-Difference and Resistance

Barlow <sup>(44)</sup>, 1903, developed a relation between the thermomagnetic longitudinal potential difference and the change in resistance in a magnetic field in the following way.

The longitudinal difference of potential, set up by a magnetic field between two points at a distance  $dl$  apart and whose difference in temperature is  $dt$ , is:

$$E = -n dt + (r - r_0) i dl,$$

$$\text{or} \quad E = -n \frac{dt}{dl} \cdot dl + (r - r_0) i dl.$$

If the heat and electric currents through the plate are

so adjusted in magnitude and direction that the galvanomagnetic and thermomagnetic E. M. F.'s are equal and opposite, then  $E = 0$ , and we have

$$\frac{n}{r - r_0} = i \div \frac{dt}{dl},$$

in which  $n$  is the longitudinal thermomagnetic effect, and  $r$  and  $r_0$  the specific electric resistances in and out of the magnetic field,  $i$  the current density in the plate, and  $\frac{dt}{dl}$  the temperature gradient.

## CHAPTER XV

### THERMOMAGNETIC LONGITUDINAL TEMPERATURE DIFFERENCE

#### 1. Thermal Conductivity in a Magnetic Field

A substance conducting a heat current, placed in a magnetic field  $H$ , Fig. 125, exhibits a change in temperature at points  $D$  and  $E$ , along the flow of heat. This effect is in reality a change in the thermal conductivity of the substance.

The coefficient of the effect may be represented by

$${}_hL_h = \frac{\Delta T}{l} \div \frac{dt}{dl} H,$$

in which  $l$  is the distance between

$D$  and  $E$ , and  $\frac{dt}{dl}$  is the temperature gradient. If  $t_2$  and  $t_1$  are the temperatures at  $D$  and  $E$ ,

respectively, then, approximately,

$${}_hL_h = \Delta T \div H(t_2 - t_1).$$

Maggi<sup>(353)</sup>, 1850, who employed Sénarmont's<sup>(477)</sup> method of melting wax on the plate, thought that the thermal conductivity of *iron* was increased by the magnetic field along the equatorial direction and decreased along the axial direction.

Holmgren<sup>(263)</sup>, 1862, found no change in the thermal

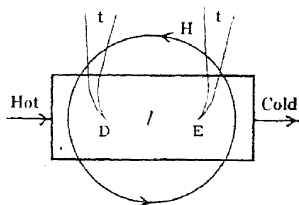


Fig. 125.

conductivity of *iron* in a magnetic field, and claimed that Maggi's results were due to experimental error.

Matteucci <sup>(361)</sup>, 1864, confirmed Holmgren's observations; and Naccari and Bellati <sup>(383)</sup>, 1877, who used thermo-couples as well as Sénarmont's method, could detect no change in the conductivity in a magnetic field.

Tomlinson <sup>(539)</sup>, 1878, employed the method of Despretz, and found a change in the conductivity in *iron* and *steel* in a magnetic field. Tomlinson's observations were not confirmed by Trowbridge and Penrose <sup>(550)</sup>, 1883.

In 1886 Battelli <sup>(46)</sup> carried out a careful investigation, with the result that longitudinal magnetization increased the thermal conductivity of *iron* about 0.2%, and a transversal magnetization reduced the conductivity about 0.04%, in a field of 1400 gauss.

Righi <sup>(447)</sup>, 1887, and Leduc <sup>(318)</sup>, 1887, determined the change in thermal conductivity in *bismuth* by means of thermo-couples, and both found a decrease in the conductivity in a magnetic field.

Nernst <sup>(387)</sup>, 1887, could discover no change in the conductivity of *bismuth* in a magnetic field. In a *copper* bar heated at one end, the other end of which was in a field of 5000 gauss, he noticed a fall in temperature in the field of from 2° to 3° C. This effect varied as the square of the field strength. Nernst did not think the thermal conductivity of the copper was altered by the field. In searching for a compensation for the cooling of the bar between the poles of the magnet, he found that on each side of the field the copper bar had increased in temperature to as much as 5° C. The same phenomenon occurred when the pole pieces were covered with lampblack or paper, to prevent reflection of the heat rays. Nernst also found that a thermometer held between the poles of the magnet showed a fall in temperature when the electromagnet was excited. Faraday <sup>(194)</sup> had also noted a similar effect of a magnetic field on *air*.

Aschkinass <sup>(19)</sup>, 1903, observed a change in temperature in a *copper* plate placed in a magnetic field. The temperature sank at points of higher intensity and rose at points of lower intensity. The change in temperature was considered to be due in large measure to the change wrought in the temperature of the adjacent *air* by the field.

Weiss and Piccard <sup>(577)</sup>, 1918, observed that, in the neighborhood of the Curie point, a magnetic field produced a rise in temperature in *nickel*, as shown in the table below. The Curie point for the nickel studied was 629.6° Abs.

627.2° Abs.		634.9° Abs.	
<i>H</i>	$\Delta T$	<i>H</i>	$\Delta T$
10,050	0.564°	10,050	0.37°
14,950	0.742°	14,950	0.57°

When the magnetic field was removed, a corresponding lowering in temperature followed.

Ettingshausen <sup>(171)</sup>, 1888, observed a decrease of 2 to 5 % in the thermal conductivity of *bismuth* in a field of 9000 gauss. The electrical conductivity in a similar field was found to decrease from 27 to 30 %.

Korda <sup>(300)</sup>, 1899, noted a diminution in the thermal conductivity of an *iron* disk along the lines of force, while no change was observed perpendicular to the field. The decrease in conductivity was proportional to the square of the field.

Van Everdingen <sup>(182)</sup>, 1899, by means of thermo-couples measured the change in thermal conductivity in *bismuth* in a transverse magnetic field of 6000 gauss, with the result that the ratio of the conductivity without the field to that within the field was found to be 1.058. Leduc's (l. c.) ratio for a field of 7800 gauss was 1.057, while that of Ettingshausen (l. c.) was 1.021 for 9000 gauss.

Schweitzer <sup>(474)</sup>, 1900, found that the decrease in thermal conductivity in *iron* was nearly proportional to the first power

of the magnetization. He employed the Sénarmont method, which seems subject to large experimental errors.

Lloyd <sup>(332)</sup>, 1901, detected a decrease of 10% in the thermal conductivity of *tellurium* in a field of 4700 gauss.

Blyth <sup>(73)</sup>, 1903, by means of thermo-couples measured the change in thermal conductivity in *bismuth*, *mild steel*, and *soft iron*. For bismuth the decrease in conductivity was less than  $\frac{1}{2}$ % in a transverse field of 3500 gauss. For mild steel the decrease was 3.3% in a transverse field of 7500 gauss. In a longitudinal field of 51 gauss the decrease was 4%. For iron in a transverse field of 7850 gauss the decrease was about 1%; while in a longitudinal field of 16,000 magnetic induction the diminution was 10.5%.

Lafay <sup>(305)</sup>, 1903, detected what he termed an appreciable diminution in the thermal conductivity in *iron* both in a transverse and longitudinal field. Lownds <sup>(345)</sup>, 1903, measured the change in thermal conductivity in a plate of *bismuth*, 1.6 cm. long, cut parallel to its principal axis. The plate was too small to use the thermo-couple method. A modification of the Sénarmont method was employed. The plate was covered with elaidic acid, as suggested by Voigt <sup>(559)</sup>. The plate was heated by a long, heavy, pointed copper wire. The point was placed at the centre of the plate. The isothermal curves formed on the plate by the melting of the wax were ellipses, whose major axes were perpendicular to the crystallographic axis, both when the plate was in and out of the transverse magnetic field, showing that the thermal conductivity was greater perpendicular to the axis. The conductivity was proportional to the squares of the axes. The following table

$H$	Ratio of Axes of Ellipse	Ratio of Thermal Conductivity	Ratio of Electrical Conductivity
0	$\frac{1}{\parallel} = 1.19$	$\frac{1}{\parallel} = 1.42$	$\frac{1}{\parallel} = 1.78$
4980	$\frac{1}{\parallel} = 1.34$	$\frac{1}{\parallel} = 1.80$	$\frac{1}{\parallel} = 1.87$

gives the ratios of the thermal and electrical conductivities in the two directions, without and within the magnetic field.

Schmaltz <sup>(472)</sup>, 1905, employed Röntgen's <sup>(455)</sup> modification of Sénarmont's method, and found that in a transverse field of about 1200 gauss the thermal conductivity of *nickel* suffered a decrease of nearly 5 % in the axial direction, as compared with the equatorial. According to Bordoni <sup>(91)</sup>, 1910, the magnetic field reduced the thermal conductivity of *iron* more than that of *steel*.

Hall and Campbell <sup>(247)</sup>, 1911, found that a transverse magnetic field apparently decreased the longitudinal temperature-gradient in soft *iron*.

Wold <sup>(604)</sup>, 1916, measured the thermal conductivity of *tellurium* by the King method, both in and out of the magnetic field. In a field of 6650 gauss the decrease in conductivity was 19 %. If we substitute this value in the equation

$$\frac{\Delta k}{k} = A H^2,$$

we shall have

$$A = 43 \times 10^{-8}$$

Lloyd's (l.c.) observations gave

$$A = 45 \times 10^{-8}, \text{ for } \textit{tellurium}.$$

## 2. Theory of the Change of Thermal Conductivity in a Magnetic Field.

Since the change in the thermal conductivity of a metal in a magnetic field is not reversed on the reversal of the field, the effect should vary as the square of the field strength, and in general the effect would be represented by the equation,

$$\frac{\Delta k}{k} = A H^2,$$

where  $\Delta k$  is the change in conductivity,  $k$  the conductivity in zero field,  $H$  the field strength, and  $A$  a constant depending on the metal.

Experimentally it has been found that this equation agrees approximately with the observed results, in most cases.

Livens <sup>(329)</sup>, 1915, starting with the usual assumptions of the

free electron theory, deduced the following expression for the change of thermal conductivity in a transverse magnetic field:

$$\frac{\Delta k}{k} = \frac{v^2 l_m^2}{q^4 S - 1} \left[ \left( 1 + \frac{2}{S} \right) \frac{\Gamma\left(2 + \frac{6}{S}\right)}{\Gamma\left(3 + \frac{2}{S}\right)} - \frac{\left(\frac{1}{2} - \frac{2}{S}\right)\left(\frac{3}{2} + \frac{4}{S}\right)}{\left(2 + \frac{2}{S}\right)} \left( \frac{\Gamma\left(\frac{3}{2} + \frac{4}{S}\right)}{\Gamma\left(2 + \frac{2}{S}\right)} \right)^2 \right]$$

The quantities above have their usual significance, while

$$v = \frac{eH}{mc}.$$

According to the above expression,  $\frac{\Delta k}{k}$  should always have a positive value, no matter what value  $S$  may have; that is, the thermal conductivity of all substances should increase in the magnetic field.

As already seen in the foregoing pages, the change in thermal conductivity in the few metals thus far examined has been a decrease.

Livens thinks that this lack of agreement of sign may be accounted for by remembering that the effective field inside the metal is made up of the impressed field and a local field set up by the molecules or molecular groups, and that in some cases this local field may reverse the sign of the effective field. The effect has been measured only in the ferromagnetic metals, *iron* and *nickel*, and in the complex and extraordinary metals *bismuth* and *tellurium*, in which the local field may play a major part. It remains to be seen whether such metals as *gold*, *silver* and *copper* will give the sign called for by Livens' equation.

## CHAPTER XVI

### 1. Influence of Magnetism on Thermoelectric Power of Metals.

Since the influence of magnetism on the thermoelectric power of metals has a direct or indirect bearing on the several effects already treated, it seems worth while to give an outline of the work already done on this subject.

In 1856 Sir Wm. Thomson <sup>(533)</sup> observed that magnetism changed the thermoelectric property of a metal. He found that in a couple made of magnetized and unmagnetized *iron*, a current of electricity flowed through the hot junction from unmagnetized to magnetized iron, when magnetized longitudinally.

When the *iron* was magnetized transversally the thermoelectric current flowed in the opposite direction.

In the case of longitudinally magnetized *nickel* the thermo-E. M. F. was in the opposite direction to that set up in longitudinally magnetized *iron*. The behavior of *nickel* in a transverse field seems not to have been determined by Thomson.

Strouhal and Barus <sup>(512)</sup>, 1881, measured the change in thermo-electric power of magnetized and unmagnetized *iron* against *copper*, with the following results:

Metal	$t_1$	$t_2$	E. M. F.
Iron (mag.) . . . .	16.5° C.	99.2° C.	$8.30 \times 10^{-8}$
Iron (unmag.) . .	„	„	8.26 „

The magnetization thus increased the thermoelectric power of iron with respect to copper.

Ewing <sup>(191)</sup>, 1886, observed a variation of the thermoelectric power of *iron* under stress, both when magnetized and not magnetized. The variation exhibited hysteresis similar to that of the magnetic properties of iron. Ewing, however, attributed the change in thermo-E. M. F., not to magnetization, but to an irreversible change in molecular structure brought about by the stress.

Grimaldi <sup>(222)</sup>, 1887, examined the behavior of a *bismuth-copper* thermo-couple in a transverse magnetic field. *Commercial bismuth* showed a decrease in the thermo-E. M. F., while *pure bismuth* exhibited an increase. In longitudinal fields the change in thermo-E. M. F. had the same sign as in transverse fields, but was smaller.



If  $E_0$  is the thermo-E. M. F. in zero field, and  $E$  that in the magnetic field, then the ratio of change is

$$\delta = \frac{E_0 - E}{E_0}.$$

For *impure bismuth* then  $\delta$  was positive, while for the *pure metal*  $\delta$  was negative.

Bachmetjew <sup>(33)</sup>, 1891, found that magnetization of *iron* and *nickel* produced changes in their thermoelectric properties similar to those brought about by elongation of the metal wires. He attributed the change in property when magnetized to the elongation due to magnetization, and not to magnetization *per se*.

According to Chassagny <sup>(115)</sup>, 1893, a longitudinal field increased the thermo-E. M. F. of an *iron-copper* couple to a maximum at 55 gauss; the E. M. F. then decreased in higher fields, up to 200 gauss.

The thermoelectric power of *iron* and *nickel* in a magnetic field was investigated by Battelli <sup>(47)</sup> in 1893.

In a transverse magnetic field the thermo-E. M. F. of *iron-copper* couple was found to increase in fields up to 15,000 gauss, and then approached saturation. The increase was greater at higher temperatures. In a longitudinal field the *Fe-Cu* couple in fields up to 18,000 gauss exhibited an increase in thermo-E. M. F. similar to that found in the transverse field.

The thermo-E. M. F. of a *nickel-copper* couple increased in a transverse field up to 12,000 gauss, reached a maximum, and then fell off slightly. In longitudinal fields up to 12,500 gauss the E. M. F. of a *Ni-Cu* couple increased and approached saturation.

In both transverse and longitudinal fields a couple of *Fe* <sub>(mag.)</sub> vs. *Fe* <sub>(non-mag.)</sub> was thermoelectrically positive, while in the same fields a couple of *Ni* <sub>(mag.)</sub> vs. *Ni* <sub>(non-mag.)</sub> was thermoelectrically negative.

Bidwell <sup>(66)</sup> found both *nickel* and *iron* couples, as above

above defined, thermoelectrically positive. The presence of iron in the nickel may account for this.

Houllevigue <sup>(268)</sup>, 1896, measured the change in thermo-E.M.F. of *iron-copper* and *steel-copper* couples in a magnetic field. In the case of the iron-copper couple, the E.M.F. first increased rapidly, reached a maximum, and then decreased and became zero at about 350 gauss, as indicated in Fig. 126, in which  $\Delta E$  is given in microvolts.

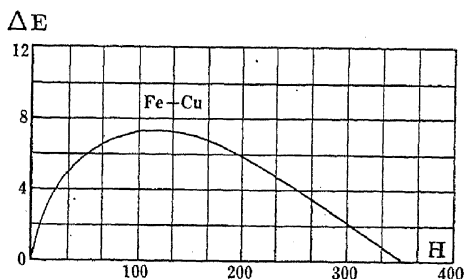


Fig. 126.

Houllevigue obtained this equation for the change of thermo-E.M.F. in *iron*, with change of field and temperature,

$$\Delta E = 10^{-7} \left[ 125(t_2 - t_1) + 0.508(t_2^2 - t_1^2) \right] \frac{H(350 - H)}{1 + 0.0428H}.$$

Accordingly  $\Delta E$  changes sign when  $H$  is greater than 350.

In the case of a *steel-copper* couple, a longitudinal magnetic field, up to 1200 gauss, always produced a decrease in the thermo-E.M.F., and this decrease was greater the higher the mean temperature of the couple.

The effect in a transverse field for the same couple was less marked, but of the same sign.

Defregger <sup>(141)</sup>, 1897, measured the change of thermo-E.M.F. in a *crystalline* plate of *bismuth* in fields from 2000 to 21,000 gauss. The following equation represented the change:

$$\frac{\Delta E}{E_0} = 0.003392 + 0.0000102 H.$$

For a homogeneous *crystal of bismuth*, with heat-flow parallel to the principal axis, this equation held:

$$\frac{\Delta E}{E_0} = 0.001050 + 0.00001093 H - 0.000000000000449 H^2.$$

When the heat-flow was perpendicular to the crystal axis, the change in E. M. F. was about  $1/10$  to  $1/60$  that when the flow was parallel to the axis.

Lownds<sup>(344)</sup>, 1901, gave the following comparison between his determination of the change of thermo-E. M. F. and that of Defregger (l. c.) for a *crystal of bismuth* in a magnetic field.

Crystal Axis	$H$	Defregger	Lownds
to Heat Flow .	2375	2.26 %	2.55 %
„	6100	6.95	8.54

Rhoads<sup>(431)</sup>, 1902, has shown that the thermoelectric power of *iron against copper* increased rapidly to a maximum in a magnetic field, and then diminished on further increase in the field, became zero at about 400 gauss, then changed sign. The variation of the thermo-E. M. F. was similar to that shown in the curve of Houllévigüe, Fig. 126.

For *nickel* the thermo-E. M. F. decreased rapidly at first, and then, on further increase in field, seemed to approach a saturation value.

In the case of *iron* the increase in the thermo-E. M. F. curve was analogous to the elongation curve due to magnetization; while the decrease of the thermo-E. M. F. for *nickel* was similar to the contraction curve of the metal in a magnetic field.

Van Aubel<sup>(27)</sup>, 1903, examined the change in thermoelectric power of pure *bismuth*, and its *alloys with lead*, in the magnetic field. In a longitudinal field pure bismuth increased in its thermoelectric height. Traces of lead caused a decrease in the thermo-E. M. F., and further addition of lead reduced the E. M. F. below that of bismuth without the field.

In a transverse field additions of lead increased the thermo-E.M.F. without any change of sign.

The change in the thermoelectric power of *iron*, *nickel*, and *cobalt* in a magnetic field, was determined by Bidwell<sup>(66)</sup>, 1904. The thermo-E.M.F. in some specimens of *iron* in a longitudinal field increased rapidly and reached a maximum between 100 and 200 gauss, then fell off, but did not reverse in fields from 1000 to 1500 gauss; while in other specimens the thermo-E.M.F. changed sign in fields from 200 to 500 gauss. The curves for the variation of the thermo-E.M.F. with field resembled in general those for the change of length in the magnetic field.

The thermo-E.M.F. of *nickel* in the magnetic field decreased rapidly and soon reached a constant value. The curve for the contraction of nickel in the magnetic field closely resembled that for the change of thermo-E.M.F.

The thermo-E.M.F. of *cobalt* increased in the magnetic field and approached saturation values in fields from 1000 to 1500 gauss. The change in the thermo-E.M.F. of cobalt seemed to bear no definite relation to the magnetostriction of the metal.

Both magnetized *iron* and *nickel* were found to be thermoelectrically positive with respect to the unmagnetized metal.

Posejpal<sup>(420)</sup>, 1909, found that couples of *Fe-Zn* and *Fe-Cu*, placed with the warmer junction in fields of from 4,500 to 10,000 gauss, exhibited an E.M.F. of magnetization which was opposed to the purely thermo-E.M.F. When the cooler junction was in the field, the two E.M.F.'s were of the same sign. A *Cu-Zn* couple gave an E.M.F. in the field just the opposite to that of the other two couples.

Posejpal maintained that the E.M.F. of magnetization was not simply a change in the thermo-E.M.F., but an independent E.M.F. He, however, stated that his experiments were not sufficient to show whether this E.M.F. of mag-

netization existed when the couples were absolutely at the same temperature throughout.

The influence of the magnetic field on the thermoelectric property of *iron* and *steel* was observed by Cohen <sup>(123)</sup>, 1909, with these results:

1. For *soft iron* the magnetic field produced an increase in the thermo-E. M. F.

2. For *hammered steel* the magnetic field produced a diminution in the thermo-E. M. F.; for *tempered steel*, an increase; while for *annealed steel*, an increase in weak fields, and a decrease in fields above 1000 gauss.

3. In all cases the influence of the field disappeared at the temperature of the magnetic transformation of the metal.

Grondahl and Karrer <sup>(227)</sup>, 1911, observed that the thermo-E. M. F. of *iron* in a longitudinal field increased to a maximum between 300 and 400 gauss, then decreased, but in fields up to 3000 gauss there was no reversal.

The thermo-E. M. F. of *nickel* and *cobalt* diminished rapidly and soon approached saturation values.

In couples of *Al-Cu* and *Zn-Cu* no change in thermo-E. M. F. as great as  $10^{-7}$  volts could be detected.

Bordoni <sup>(92)</sup>, 1911, examined the change of thermo-E. M. F. of *iron*, *steel*, and *nickel* in a magnetic field. Against the ferromagnetic metals he used *wires* of *copper*, *silver*, *manganese*, and *zinc*, and found that the change in thermo-E. M. F. was independent of the nature of the metal joined to the magnetic metals, as Houllevigue <sup>(268)</sup> had found before.

In a longitudinal field, up to 1500 gauss, the thermo-E. M. F. of *iron* reached a maximum, then fell off, the curve indicating that the effect would reverse sign at somewhat higher fields. The maxima increased with the temperature. In the case of some of the curves for steel, after the maximum, there was a reversal of sign.

For *nickel* the thermo-E. M. F. diminished rapidly in a longitudinal field, and approached saturation values. The

decrease was greater for higher temperatures. Magnetized *iron* and *nickel* were both thermoelectrically positive with respect to the unmagnetized metal, while magnetized *tempered steel* was negative with respect to non-magnetized steel.

Königsberger and Gottstein <sup>(297)</sup>, 1915, measured the change in thermo-E. M. F. of *silicon against iron* in a magnetic field of 5800 gauss, with the result that for one specimen

$$\frac{\Delta E}{E \cdot H^2} = + 3.5 \times 10^{-11},$$

and for another,

$$\frac{\Delta E}{E \cdot H^2} = + 1.2 \times 10^{-11}.$$

An increase in each case. This change in thermo-E. M. F. was found to be almost numerically equal to the increase in resistance, or

$$\frac{\Delta E}{E} = \frac{\Delta r}{r} \quad (\text{nearly}).$$

The two ratios were

$$\frac{3.5}{1.2} = \frac{2.6}{1.0} \quad (\text{nearly}).$$

Since the *silicon* was joined to *iron*, and since others have claimed that the change in thermo-E. M. F. is due solely to the action of the magnetic field on the magnetic metal, probably the above change in E. M. F. should be attributed to the iron and not to silicon.

The following expression for the change in thermoelectric force in a magnetic field was developed by La Rosa <sup>(457)</sup>, 1919:

$$P_H - P_0 = \frac{2}{3} \frac{\alpha}{e} \log \left\{ \left[ 1 + \frac{1}{6} \left( \frac{e l_0}{2 m v} \right)^2 H^2 \right] \frac{q_H}{q_0} \right\},$$

in which  $P_H$  and  $P_0$  are thermoelectric power,  $q_H$  and  $q_0$  the specific resistance, respectively in and out of the magnetic field, and  $l_0$  the mean free path of an electron in zero field. The other letters have their usual significance.

La Rosa <sup>(458)</sup>, 1919, measured the change in thermoelectric power and resistance of *bismuth* in a magnetic field, with the results given in the table below.

$H$	$\frac{\Delta P}{P_0}$	$\frac{\Delta r}{r}$	$H$	$\frac{\Delta P}{P_0}$	$\frac{\Delta r}{r}$
1000	0.015	—	5100	0.115	0.136
1600	0.030	—	5900	0.127	0.172
2100	0.044	0.030	6600	0.137	0.207
3150	0.067	0.061	7600	0.149	0.253
3700	0.080	0.082	9100	0.154	0.322
4500	0.102	0.114	10100	0.157	0.387

The relation here between  $\frac{\Delta P}{P_0}$  and  $\frac{\Delta r}{r}$  is a close one, and points to some intimate connection between the two changes in a magnetic field.

## 2. Thermo-E. M. F. of Alloys in a Magnetic Field

Spadavecchia<sup>(501)</sup>, 1899, studied the influence of the magnetic field on the thermoelectric property of *bismuth* and its *alloys* with *tin* and *lead*. The addition of a small amount of tin to the bismuth caused the thermo-E. M. F. of a Cu-Bi-Sn couple to diminish in a constant field. Further addition of tin caused the E. M. F. to change sign and undergo several irregular variations.

When small quantities of lead were added to the bismuth of a Cu-Bi-Pb couple, the thermo-E. M. F. in a constant field increased. Subsequent additions of lead caused a change in sign and irregular variations in the E. M. F.

Guthe and Austin<sup>(231)</sup>, 1906, sought for a change in the thermoelectric property of *Heusler alloy* in fields up to 1000 gauss, and at temperatures from 0° to 100° C. They concluded that, if there were any change in the thermoelectric property, it was less than 0.5 microvolt.

Grondahl and Karrer<sup>(227)</sup>, 1911, however, found that a longitudinal magnetic field changed the thermoelectric property of the *Heusler alloys* of varying composition. In several of the alloys the magnetic field caused a steady increase in the thermo-E. M. F., with no decided tendency to saturation values. In several other specimens there was an initial decrease in

the E. M. F., followed by a reversal in sign, and then a continued increase. In all cases, above weak fields, the change in thermo-E. M. F. was more nearly linear than is the case in the ferromagnetic metals.

Grondahl <sup>(226)</sup>, 1914, endeavored to find a relation between the change in thermo-E. M. F. in a longitudinal magnetic field and magnetostriction in *Heusler alloys*. The  $\frac{\Delta l}{l}$  curves all increased rapidly and approached saturation in moderately high fields; while some of the thermo-E. M. F. curves were positive and almost linear, and others passed through a small negative maximum, reversed sign, and then became linear. The magnetostriction decreased with rise in temperature, and disappeared at the transformation range; while the effect of the magnetic field on the thermo-E. M. F. changed with rise in temperature, but did not disappear at temperatures well above the transformation range.

### 3. Change of the Peltier Effect in a Magnetic Field

The change of the Peltier effect in a magnetic field is intimately connected with the change of thermoelectric power in the field. The equation for the Peltier effect is

$$\Pi = T \frac{\partial E}{\partial T},$$

where  $\frac{\partial E}{\partial T}$  is the thermoelectric power of the metal outside the field. Then in the magnetic field

$$\Pi' = T \frac{\partial E'}{\partial T},$$

and

$$\Pi' - \Pi = T \frac{\partial (E' - E)}{\partial T},$$

where  $E$  and  $E'$  are respectively the thermo-E. M. F.'s of the couple outside and within the magnetic field, and  $\partial T$  may be taken as  $(t_2 - t_1)$ , where  $t_2$  and  $t_1$  are the temperatures at the junctions; and  $T$  is in absolute temperature.

Battelli <sup>(47)</sup>, 1893, has shown that the Peltier effect in *iron*



and in *nickel* is modified by the magnetic field. In a transverse field the Peltier effect in an *iron-copper* couple increased, rapidly at first, then less rapidly, as the field increased, followed by an approach to a saturated value.

The increase in a *nickel-copper* couple was less than in the iron.

In a longitudinal field the Peltier effect was increased both in *iron* and in *nickel*. The variation with the field was similar to that in a transverse field, but the change was greater, in the case of iron 2.5 times as great, in nickel 1.5 times as great.

Houllevigue<sup>(268)</sup>, 1896, measured the change in Peltier effect in *iron* and *steel* in a magnetic field. In the case of *iron* the change in the effect increased to a maximum at about 100 gauss, then decreased and changed sign at about 350 gauss, as shown in Fig. 127, where  $100 \frac{\Pi'}{\Pi}$  is plotted against the field.

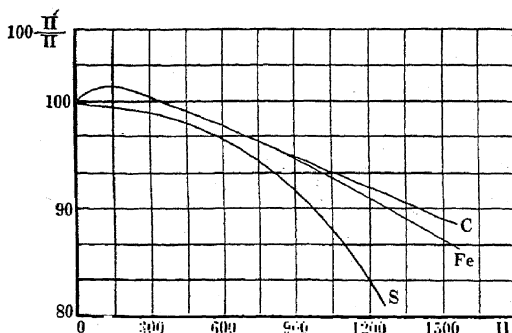


Fig. 127.

From the equation

$$\Pi' - \Pi = \frac{T}{J} \frac{\partial (E' - E)}{\partial T},$$

and the empirical equation for  $\Delta E$  or  $(E' - E)$ , p. 261, Houllevigue deduced this equation for *iron*:

$$\Pi' - \Pi = -10^{-12} \cdot \frac{T + 273}{4.17} (1.25 + 1.016 T) \left\{ \frac{H(350 - H)}{1 + 0.0428 H} \right\}.$$

He finds this equation gives values in close agreement with those observed, as shown by curve *C*, Fig. 127.

The Peltier effect in *soft steel*, *S*, as shown in Fig. 127, decreases more and more rapidly as the field increases.

The curves in Fig. 128 show that the maximum increase in the Peltier effect in *iron* in the magnetic field increased with the temperature.

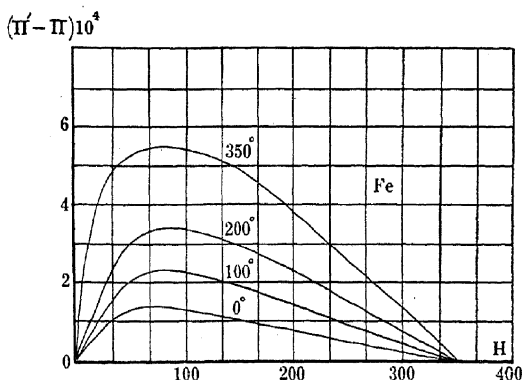


Fig. 128.

Pochettino <sup>(417)</sup>, 1899, investigated the change in the Peltier effect of an *iron-copper* couple in a longitudinal magnetic field. The effect increased to a maximum in a field of about 98 gauss, decreased, and passed through its normal value at about 345 gauss. These results are confirmatory of Houllevigue's observations.

The change in the Peltier effect of *bismuth* against *copper* in a magnetic field was measured by Borelius and Lindh <sup>(95)</sup>, 1917. The change produced by a field of 2300 gauss varied from  $\Delta \Pi = 0.042 \times 10^{-8}$  cal./coulomb, to  $\Delta \Pi = 0.0237$  " " according to the direction of the current and of the field with respect to the crystal axis of the *bismuth*.

## ADDENDUM.

### HALL'S LATEST THEORY OF THE HALL EFFECT

In his latest theory of the Hall effect, Hall <sup>(246a)</sup>, 1923, postulated the presence of two transverse streams of electrons in a metal plate placed in a magnetic field. One of the streams consists of the free electrons, which gives rise to a negative Hall effect,  $R_f$ ; while the other stream consists of associated electrons, and produces a positive effect,  $R_a$ . One or the other of these effects will generally predominate, according to the kind of metal, its physical condition and temperature, and the strength of the magnetic field.

The expression for the free electron effect is given as:

$$R_f = - (6.28 d' \cdot x) \div k T^{0.5},$$

in which  $d' \cdot x$  is the mean free path of the electrons at temperature  $T$ ,  $x = \lambda_0 n^{1/2}$ ,  $\lambda_0$  the mean free path at  $0^\circ \text{C}$ .,  $n$  the number of atoms per c.c., and  $k$  the specific conductivity, which is equal to the sum of the partial conductivities,  $k_a$  and  $k_f$ .

The observed Hall effect,  $R$ , is given by the equation:

$$R = R_a \cdot \frac{k_a}{k} + R_f \cdot \frac{k_f}{k}.$$

From this relation  $R_a$  may be computed.

Taking into account the parts played by the two kinds of electrons, Hall deduced certain definite relations that should exist between the Hall and Ettingshausen effects, between the Hall and Nernst effects, and between the Nernst and Righi-Leduc effects.

# APPENDIX

## ELECTRONS AND ATOMS

While it is not the province of this treatise to discuss the electronic structure of the atom, nor to take up the various electron theories of heat, electricity, and magnetism, yet it seems worth while to point to some works on these subjects.

It is probable that satisfactory explanations of the phenomena treated in this monograph cannot be had, until the internal structure of the atom and the interaction of atom and electron are better understood.

To this end the works of the following authors are referred to in the Bibliography:

### 1. On Electronic Structure of the Atom

Larmor <sup>(309)</sup>, 1900; Kelvin <sup>(535)</sup>, 1902; Abraham <sup>(2)</sup>, 1903; Thomson (Sir J. J.) <sup>(528-9)</sup>, 1904—1914; Lodge <sup>(333)</sup>, 1903, 1906; Rutherford <sup>(464-5)</sup>, 1905—1920; Bohr <sup>(75-6)</sup>, 1911—1913; Barkla <sup>(42)</sup>, 1911; Broek <sup>(100-1)</sup>, 1913; Moseley <sup>(382)</sup>, 1913; Nicholson <sup>(388)</sup>, 1914; Herzfeld <sup>(258)</sup>, 1915; Kossell <sup>(301)</sup>, 1916; Lewis <sup>(326)</sup>, 1916; Langmuir <sup>(308)</sup>, 1919; Crehore <sup>(134-5)</sup>, 1917—1920; Rutherford <sup>(464-5)</sup>, 1911—1920; Sommerfeld <sup>(500)</sup>, 1921.

### 2. On Electronic Conduction

Riecke <sup>(436-442)</sup>, 1881—1915; Voigt <sup>(560-2)</sup>, 1898—1902; Drude <sup>(149-52)</sup>, 1900—1905; Lorentz <sup>(339-40)</sup>, 1903, 1905; Gans <sup>(208)</sup>, 1906; Weiss <sup>(573-6)</sup>, 1907—1911; Jeans <sup>(276)</sup>, 1909; Thomson (Sir J. J.) <sup>(520, 1, 6, 7)</sup>, 1900—1915; Zahn <sup>(613)</sup>, 1913, 1914; Richardson <sup>(432-4)</sup>, 1912—1915; Hall <sup>(244, 6)</sup>, 1914, 1917; Borelius <sup>(93)</sup>, 1919.

Extended references to works on electrons and electronic theories may be found in *Ions, Électrons, Corpuseles*, 2 vols., Paris, 1905; and in *Jahrbuch der Radioaktivität und Elektronik*, Leipzig, 1904—1923.



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